

中國文化大學 100 學年度轉學招生考試

系組：物理學系三年級

日期節次：7月26日第2節 11:00-12:20

科目：力學 (11-75)

1. A particle of mass m moves in the $x-y$ plane so that its position vector is $\vec{r} = r \cos(\omega t) \hat{i} + r \sin(\omega t) \hat{j}$, where r and ω are constants. Find the velocity and the acceleration. (15%)
2. The position of a particle is given by $\vec{r} = A(e^{\alpha t} \hat{i} + e^{-\alpha t} \hat{j})$, where α is a constant. Find the velocity and the acceleration. (15%)
3. A particle of mass m moves in the $x-y$ plane so that its position vector is $\vec{r} = a \cos(\omega t) \hat{i} + b \sin(\omega t) \hat{j}$, where a , b and ω are constants and $a > b$. (a) Show that the particle moves in an ellipse. (b) Show that the force acting on the particle is conservative. (20%)
4. Show by means of the substitution $r = \frac{1}{u}$ that the differential equation $m(\ddot{r} - r\dot{\theta}^2) = f(r)$ for the path of the particle in a central field can be written as

$$\frac{d^2 u}{d\theta^2} + u = -\frac{1}{ml^2 u^2} f\left(\frac{1}{u}\right). \quad (20\%)$$

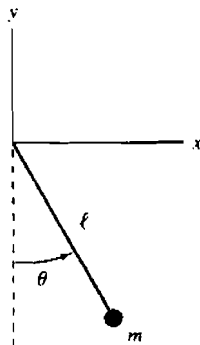


Fig.-1

5. As shown in Figure-1, use the (x,y) coordinate system to find (a) the kinetic energy T , the potential energy U , and the Lagrangian L for the case of a simple pendulum (with length l , mass m) which is moving in a uniform gravitational field. Using Lagrangian formalism to find the equation of motion (b) in the (x,y) coordinate and (c) in the polar coordinate (r,θ) . (30%)