

題目中,  $A, B$  皆代表矩陣.  $A^T$  代表  $A$  的轉置矩陣.  $A^{-1}$  代表  $A$  的反矩陣.

1~6 題為簡答題, 每題 5 分, 只須回答 True 或 False :

1. Let  $A$  and  $B$  be 2 matrices, then  $(A+B)^2 = A^2 + 2AB + B^2$ .
2. If  $A$  is invertible and  $AB = AC$ , then  $B = C$ .
3. If  $A$  is not symmetric, then  $A^{-1}$  is not symmetric.
4. The columns of a matrix are a basis for the column space.
5.  $AB$  and  $BA$  have the same determinant.
6. Every positive definite matrix is invertible.

7~13 題為計算證明題, 每題 10 分, 必須寫清楚每一題的詳細過程 :

7. Let  $A$  be a 2 by 2 matrix such that  $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ . Find  $A$ .

8. There are sixteen 2 by 2 matrices whose entries are 1's and 0's. How many of them are invertible?
9. A permutation matrix has the rows of identity matrix  $I$  in any order. Find a 3 by 3 permutation matrix  $P$  with  $P^3 = I$  but  $P \neq I$ .

10. Let  $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix}$ . Find the rank of  $AB$ .

11. A 3 by 3 matrix  $B$  is known to have eigenvalues 0, 1, 2. Find the determinant of  $B^T B$ .

12. Prove that  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$  are similar.

13. Suppose a linear  $T$  transforms  $(1, 1)$  to  $(2, 2)$  and  $(2, 0)$  to  $(0, 0)$ . Find  $T(v)$  when  $v = (-1, 1)$ .