中國文化大學 99 學年度轉學招生考試

系組：應用數學系三年級　　日期節次：7 月 27 日第 3 節 13:30-14:50

科目：線性代數　(19-21)

試題中：A, B, C 皆代表矩陣. A^T 代表 A 的轉置矩陣. A^{-1} 代表 A 的反矩陣.

1~6 題爲簡答題，每題 5 分，只須回答 True 或 False：

1. If $AB = I$ and $BC = I$, then $A = C$.

2. If A is 3 by 4 and B is 4 by 5, then $(AB)^T$ is 5 by 3.

3. Let A and B be 2 matrices, then $(A-B)^2 = A^2 - 2AB + B^2$.

4. If A and B are symmetric, then the transpose of AB is BA.

5. A and $-A$ always have the same reduced echelon form.

6. If A and B are permutation matrices, then $AB = BA$.

7~13 題爲計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

7. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Find A^3.

8. Let A and B be symmetric. Prove ABA is also symmetric.

9. Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$. Find the rank of A.

10. Let $A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 2 & 6 & 4 & 8 \end{bmatrix}$. Find the dimension of the row space of A.

11. Let $A = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$. If $B^{-1}AB$ is a diagonal matrix, find B.

12. Prove: If A and B are positive definite, then $A+B$ is also positive definite.

13. Prove or disprove: $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$.