

中國文化大學 100 學年度轉學招生考試

系組：應用數學系三年級 日期節次：7 月 26 日第 2 節 11:00-12:20

科目：高等微積分 (112-74)

1.(15%) True or False:

- a) \mathbf{Q} is dense in \mathbf{R} .
- b) \mathbf{R} is complete.
- c) Every single point in an Euclidean n -space is a closed set.
- d) A connected set is also path-connected.
- e) Every integrable function is continuous.

2.(15%) Determine which of the following are open, closed, or compact in \mathbf{R}^2 :

$$A = \{(x, y) \mid 1 < x < 2 \ \& \ 2 < y < 3\}$$

$$B = \{(x, y) \mid x^2 + y^2 \leq 1\} \cup \{(0, 2), (2, 1)\}$$

$$C = \{(x, y) \mid x \in \mathbf{Z} \ \& \ y \in \mathbf{Z}\}$$

3.(15%) Show that a set A in a metric space M is closed if and only if the accumulation points of A belongs to A .

4.(10%) Show that if d is a metric on a set M , and ρ is defined by

$$\rho(x, y) = \frac{d(x, y)}{1 + d(x, y)},$$

then ρ is also a metric on M .

5.(15%) Let $f_n : \mathbf{R} \rightarrow \mathbf{R}$ be uniformly continuous and let f_n converge uniformly to f . Must f be uniformly continuous?

6.(15%) Let

$$f_n(x) = \frac{x}{1 + nx^2}.$$

Show that f_n converges uniformly on \mathbf{R} .

7.(15%) Let

$$f(x, y) = \frac{x^2 y^2}{\sqrt{x^2 + y^2}}, \quad (x, y) \neq (0, 0)$$

and

$$f(x, y) = 0, \quad (x, y) = (0, 0).$$

Is f differentiable at $(0, 0)$?