行政院國家科學委員會補助專題研究計畫成果報告

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※ **在光固體中應用有效質量法於光子與激子** ※
※ **的作用研究** ※
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計畫主持人：程思誠
共同主持人：謝文峰

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主持人：程思誠　中國文化大學物理系
共同主持人：謝文峰　交通大學光電工程研究所
計畫參與人員：陳世萍　中國文化大學物理系

一、中文摘要

利用有效質量法吾人可以了解電磁波在光晶體內傳遞的特性，經由研究電磁波在光能隙附近的傳遞，吾人發現電磁波的傳遞可以由具慣性質量的質點運動來描述，此質點運動相位具有奇異點存在，奇異點引起光漩渦的發生，進而使質點具有角動量，質點具有右旋角動量稱為帶有正電荷的光漩渦，反之則為帶負電荷的光漩渦，質點屬於電磁波於空氣能帶的傳遞為帶有正電荷，質點屬於電磁波於介電能帶的傳遞為帶有負電荷。

關鍵詞：慣性質量、光漩渦、角動量

Abstract

Using the $\mathbf{k}\cdot\mathbf{p}$ theory to study photons in a photonic crystal, we found the incident photon excites a quasi-particle (QP) of photon from the periodic background field. This QP contains an inertial mass, which is due to energy-storing mechanism occurring in the photonic crystals. This QP has phase singularity and forms an optical vortex that carries angular momentum. An optical vortex with right-handed (left-handed) angular momentum is called positively (negatively) charged. The QP of air band is positively charged and negatively charged in the dielectric band.

Keywords: Inertial mass, Optical vortex, Angular momentum.

二、緣由與目的

Bayindir et al. [1] and Ohtaka and Tanabe [2] observed “heavy photons” at coupled-cavity waveguide band edges and the fcc array of dielectric spheres. The photonic band has flat band with very slow group velocity with the features quite similar to heavy fermions. They proposed to call them as “heavy photons” or “heavily massive photons”. However, a phenomenon of slow motion of photons is not necessary to mean “heavily massive photons” unless one can show “heavy photons” have inertial or rest mass. Using the effective-mass method, Russell et al. [3] arbitrarily defined the rest mass of photon as m=¦ω/2c^2 from comparison between the Schrodinger equation of electron and Maxwell's equations of photon, where ¦ is the Planck's constant and ω is the angular frequency of photon. However, the Maxwell's equations contain relativistic effects that are absent in the Schrodinger equation. The definition of rest mass here is invalid.

Scalora et al. [5] observed large group index of an optical pulse in examining one-dimensional photonic band-edge transmission resonance and explained the results by a combination of the forward-propagating EM field of the ultrashort pulse and a quasi-standing-wave that transiently forms within the layered structure. Energy is scattered from the forward-propagating fields into the quasi-standing-wave and back into the forward-propagating fields. The wave carries and transiently storing substantial EM energy in a circulatory manner. In this report we show that the envelope function of EM wave in PBG satisfies the generalized Klein-Gordon (KG) equation. By solving the KG equation we found the incident photon excites a quasi-particle (QP) of photon from the periodic background field. This QP contains an inertial mass, which is due to energy-storing mechanism occurring in the photonic crystals. It also possesses a topological charge in the form of optical vortex that carries angular momentum. The QP of air band is positively charged and negatively charged in the dielectric band.

三、結果與討論

The appropriate EM equation for studying the photonic crystal is a hermitian equation of magnetic field $\mathbf{H}(\mathbf{r})$:

$$\nabla\times[\frac{1}{\in(\mathbf{r})}\nabla\times H(\mathbf{r})]=\frac{\mu^2}{c^2}H(\mathbf{r}),\ (1)$$

where $\in(\mathbf{r})$ is the periodic dielectric constant. We expand $\mathbf{H}(\mathbf{r})$ in terms of Kohn-Luttinger function [19] at a specific wave-vector $\mathbf{k}_0$, where the band maximum or minimum occurs. By defining P= $|\mathbf{P}|=|\mathbf{k}-\mathbf{k}_0|<<1$ near the band edge, we found the corresponding eigenvalue is $\omega_n(\mathbf{k})$ for incident wavevector $\mathbf{k}$, where n is a band index. From Notomi's discussions [6], we can define the reciprocal effective-dielectric tensor $[\frac{1}{\epsilon*}]_{\alpha\beta}$ near the band edge, where α and β are indices of three different directions of position vector $\mathbf{r}$.

Following the calculations of de Sterke and Sipe [7], we obtain a time-dependent equation of the envelope function $F_n(\mathbf{r},t)$ of Eq. (1):

$$\{(m_n c)^2 - \sum \hbar^2 [\frac{1}{\epsilon*}]_{\alpha\beta} \frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x_\beta} + \frac{\hbar^2}{c^2}\frac{\partial^2}{\partial t^2}\} F_n(\mathbf{r},t) = 0 \qquad (2)$$

This equation is the so-called generalized Klein-Gordon equation and is reduced to the Klein-Gordon equation [8] in isotropic medium. It indicates that there is an energy-storing mechanism near band edges [5]. Thus we defined $m_n=\hbar\omega_n(\mathbf{k}_0)/c^2$ as the inertial mass of quasi-particle of photon, which is dependent on the band index n and is quantized.

Note that the sign of the effective-dielectric tensor is positive near the band maximum and negative near the band minimum. We will consider the positive case of the effective-dielectric tensor and discuss the negative case later. We shall assume that the medium is effectively isotropic, thus the effective-dielectric tensor becomes a positive scalar constant and the effective speed of the light is $c*=c/\sqrt{\epsilon*}$ in the photonic crystal. We can rewrite Eq. (2) as

$$\{\frac{\hbar^2}{c*^2}\frac{\partial^2}{\partial t^2} - \hbar^2\nabla^2 + (\mu_n c*)^2\} F_n(\mathbf{r},t) = 0, \qquad (3)$$

which has been used to describe a massive spin-zero particle in the relativistic field theory [8] with $\mu_n = m_n\epsilon*$ being the effective mass of the particle. The existence of an inertial mass induces the concept of an effective mass in PBG. $\mu_n=0$ in the uniform medium, because there is only one dispersion minimum $\omega_n(\mathbf{k}_0=0)=0$ where $m_n=0$.

Because there is an energy-storing mechanism [5] near band edges, m_n can be different from zero. We can say that a particle with inertial (massive particle) is excited by the incident photon from the Bloch-type vacuum field of PBG. This massive particle is a quasi-particle (QP) of a Bloch photon, whose motion resembles a free photon having an effective index $\sqrt{\epsilon*}$ near the band gap, despite of large scattering by the crystal [6].

By applying the well-known quantization prescription, the momentum operator $\mathbf{P}=-i\hbar\nabla$ and the energy operator $E=i\hbar\frac{\partial}{\partial t}$, to Eq. (4), we obtain the relativistic energy-momentum relation $E^2=\mu_n^2c*^4+\mathbf{P}^2c*^2$ for a QP. This relation shows that a Bloch photon behaves as a free relativistic QP moving in a medium that the effective speed of light becomes c*. The relativistic effect of a free QP is consistent with the Maxwell equations, where the special relativity is satisfied. When the EM wave is entering the photonic crystal, part of the EM-wave energy is stored as the inertial mass m_n of a QP, according to the relation $m_n=\hbar\omega_n(\mathbf{k}_0)/c^2$. The rest part of the EM-wave energy becomes the kinetic energy of a QP, whose velocity $\mathbf{v}$ is not c* but $\mathbf{v}=\hbar\mathbf{P}/\mu_n$ with $|\mathbf{v}|<<c*$, since P<<1. Our theory of exciting a QP by the EM waves explains the phenomena, observed by Scalora et al. [5] and Russell et al. [3], of storage and release of EM-wave energy in PBG.

For negative effective dielectric tensor, the effective refraction index becomes imaginary. This, the photons can no longer propagating within the band. This contradicts the fact that photons do propagate near the band maximum. Such a negative dielectric tensor problem is also happened for electrons transporting on the valence band of a semiconductor. From the excitations of the background electrons on the valence band, a positively charged hole with positive effective mass is introduced as the conduction particle of the valence band in semiconductor physics[9]. Holes are moving in an opposite direction with respect to electrons. An energy for the creation of a hole is defined as the energy to take the electron from the state **P** to some zero-energy reservoir, where the energy to create an electron state is similarly defined from this same zero-energy reservoir [9].

From the electron-hole concept, we define a kind of energy to create a QP as the increased (or decreased) energy of the zero-energy reservoir when a QP is taken from this zero-energy reservoir and put into a state **P** on the air band (or a state **P**'= -**P** on the dielectric band). The effective dielectric tensor on the dielectric band is thus given absolutely positive, and the effective refraction index is always real. A QP on the dielectric band has the same behavior as a QP on the air band, except for changing in sign for the moving direction, the current and for the energy required to create the excitation. We have a sign change of the current if the envelop function $F_n(\mathbf{r},t)$ is replaced by its complex conjugate $F_n^*(\mathbf{r},t)$. $F_n(\mathbf{r},t)$ and $F_n^*(\mathbf{r},t)$ satisfy the same continuity equation [8]. We then find that $F_n(\mathbf{r},t)$ has to be a complex function in order to explain how a QP is propagating near band edges. And it is real if and only if there is no gap.

Considering the medium is effectively isotropic, we can demonstrate that solutions of Eq. (3) possess phase singularities. We used $F_n(\mathbf{r},t)=\psi(\rho,\phi,z)\exp(iPz-i\omega t)$ to describe a QP propagating along z-direction, where $\psi(\rho,\phi,z)$ is the slowly varying complex amplitude that $\partial^2\psi/\partial z^2$ is neglected, ρ is the radial coordinate, and φ is the azimuth angle. By using the "paraxial" approximation, we can simplify Eq. (3) as

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2}+2iP\frac{\partial\psi}{\partial z}=0, \quad (4)$$

where we have applied the dispersion relation, $(\hbar\omega)^2 = \mu_n^2 c^{*4} + (\hbar P c^*)^2$, of a massive QP and obtain the solution:

$$\psi(\rho,\phi,z)=\frac{\sqrt{2^{|\ell|+1}}}{\sqrt{\pi|\ell|}}\frac{\rho^{|\ell|}}{w^{|\ell|+1}}\times\exp\left(-\frac{\rho^2}{w^2}\right)\exp[i\Phi(\rho,\phi,z)],$$

where w is the transversal beam dimension, and $\psi(\rho,\phi,z)$ is normalized. The phase $\Phi(\rho,\phi,z)$ is

$$\Phi(\rho,\phi,z)=-(|\ell|+1)\arctan\left(\frac{2z}{P\rho_s^2}\right)+\frac{P\rho^2}{2R(z)}+\ell\phi,$$

where ρ_s is the beam waist parameter R(z) is the radius of the wavefront. Because $\psi(\rho,\phi,z)$ has to be a complex amplitude everywhere even at z=0, $\ell\neq 0$ and $\ell=\pm1,\pm2,\ldots$. We found $\psi(\rho,\phi,z)$ vanishes at the beam axis (ρ=0), and the phase becomes undetermined (singular). Light beams possessing phase singularities are called optical vortices [10]. Instead of separately smooth surfaces, the wavefront attains a continuous helical surface. A topological charge ℓ can be attributed to a helical wavefront, positive for the right-twirl helicoid and negative for the left twirl [10]. Phase singularity disappears when $\ell=0$, and the solution of Eq. (7) becomes an ordinary Gaussian beam. With $\ell\neq 0$, the rotation of phase around the beam axis causes a nonzero value of orbital angular momentum of a wave[10].

Therefore, a positively charged QP ($\ell>0$) and a negatively charged QP ($\ell<0$) are excited and moving in opposite directions on the air band and the dielectric band with absolutely positive effective dielectric tensor on both bands. For light refraction in PBG, Snell's law, derived from the momentum conservation law, has a different angle-sign between the air band and the dielectric band and anomalous refraction [6] near band edges can occur.

四、計畫成果自評

In our research project, we have developed a theory of EM waves propagating near photonic band gaps. The EM wave is now massive and described by the Klein-Gordon equation. By solving the KG equation we found the incident photon excites a quasi-particle (QP) of photon from the periodic background field. This QP contains an inertial mass, which is due to energy-storing mechanism occurring in the photonic crystals. It also possesses a topological charge in the form of optical vortex that carries angular momentum. The QP of air band is positively charged and negatively charged in the dielectric band. This discovery shall bring a new horizon for experimentalists to study EM waves propagating near photonic band gaps. We shall be able to study very fundamental problems that how mass, charge and particle can be created by EM waves. We shall send our results to Physical Review for publication.

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