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作者：李順發

Content（目錄）

Externality and Macroeconomic Efficiency

Shun-Fa Lee[†]

Department of Economics, Chinese Culture University

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Abstract

This paper theoretically takes many examples to demonstrate that each kind of externalities may be irrelevant to Pareto optimum even if there are no transaction costs and no coordinations across agents. Different from the existing literature, we examine this relationship at steady state in a representative-agent macroeconomic model. Externalities may imply efficiency when the distortions which made by externalities have no effect on the allocation of resources or are mutually canceled under some mild conditions.

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[†]Corresponding address: Department of Economics, Chinese Culture University, 55, Hwa-Kang Road, Yang-Ming-Shan, Taipei 1114, Taiwan. Phone: (886-2)28610511 ext. 29335; Fax: (886-2)28625229; Email: alphalee@ntu.edu.tw

1. Introduction

Externality has long been a very important topic in the economics literature. When the actions of one agent directly affect the environment of another agent, we will say that there is an externality. In general, because the agents of externalities do not have an incentive to take into account the effect of their actions on others, the outcome will be inefficient. There will be too much activity that causes negative externalities and not enough activity that creates positive externalities, relative to an optimal outcome.

However, Coase (1960) and Buchanan and Stubblebine (1962) indicate that an allocation may be classified as Pareto-optimal despite the economy with externalities. This implies that in the Pareto optimum not all externalities should be reduced to zero: we are better off keeping some of them at a positive level. There are at least two parties involved in a single externality relationship and gains from trade between the two parties arise as long as transaction costs are small enough. It follows that the externally affected party can compensate the acting party for modifying his behavior through trade, persuasion, compromise, agreement, convention, or collective action, etc.

In addition, Dahlman (1979) figures out that in the theory of externalities, transaction costs are the roots of all evil. If there are transaction costs, then it may be too costly to eliminate all externalities, so we should preserve some of them in order to reach an optimum. Alternately, if the transaction costs can be eliminated, a Pareto improvement is possible and thereby externalities are of no consequence. As a result, externalities are natural phenomena in the economy and the equilibrium of this economy may be already located at an efficient allocation. This point has significant policy implications for it suggests that the observation of externalities, taken alone, cannot provide a basis for judgment concerning the desirability of some modification in an existing state of affairs. There is not a *prima facie* case for intervention in all cases where an externality is observed to exist.

Our paper further reveals that externalities may exert efficient outcome even if no transaction costs are presented in the world and no coordinations are reached across parties. Different from the existing literature, we examine this relationship at steady state in a representative-agent macroeconomic model. This result is established when the distortions which made by externalities have no effect on the allocation of resources or are mutually canceled under some mild conditions or attainable environments. Thus, the classical argument that externalities lead to inefficiency may be violated.

We examine four types of externalities in this paper. If we dichotomize an economy into the production side and the consumption side, we can identify four types of externalities, depending on which part of the economy generates externality and which part of the economy is being affected, which include (i) consumer-to-consumption externality: consumer's activities affect utility; (ii) producer-to-consumption externality: producer's activities affect utility; (iii) producer-to-production externality: producer's activities affect production; (iv) consumer-to-production externality: consumer's activities affect production. These four types of externalities are introduced in Section 2.

In a one-sector economy with inelastic labor supply, demonstrated in Section 3, we find that producer-generated externalities as usual exert long-run inefficiency but consumer-generated externalities exert efficiency at steady state. The reason is that consumer-generated externalities play no role in determining the steady state because the steady-state capital stock and consumption is solved by the production technology alone. In fact, these kinds of externalities indeed distort the shadow value of output or the shadow price of consumption, one of endogenous variables, but this distortion has no effect on the allocation of resources.

Do producer-generated externalities really imply inefficiency? In Section 4, we find that these externalities may imply efficiency under different environments. For example, producer-to-consumption externality may exert long-run efficiency in a two-sector economy with the externality which only comes from the pure consumption good sector. If the marginal rate of technical substitution between capital and labor from the private perspective is equal to that from the social perspective, producer-to-production externality also leads to long-run efficiency in a two-sector economy with only sector-specific externality in the pure consumption good sector. In these economies, the externalities can be mutually canceled under some mild conditions. Thus, the equilibrium is still efficient though the externalities distort the allocation of resources.

Liu and Turnovsky (2005) points out that if labor supply turns into being elastic, the shadow value of output will affect the choice of leisure and hence the steady-state allocation. In this respect, consumer-to-consumption externality must imply long-run inefficiency. In Section 5, we reexamine that and find that though labor supply is elastic, the steady-state equilibrium may back to an efficient outcome in a two-consumption-goods economy under the condition that the marginal rate of substitution between two consumption goods from the private perspective is equal to that from social perspective. In this case, the externalities are also mutually canceled and then Pareto optimum remains.

Finally, some concluding remarks are outlined in the final section.

2. Types of externalities

Generally, we can classify externalities into consumption externality and production externality. In a consumption externality the utility of one consumer is directly affected by the actions of another agent. For example, some consumers may be affected by other agents' consumption of tobacco, alcohol, loud music, and so on, which are called as *consumer-to-consumption externality*. Consumers might also be adversely affected by firms who produce pollution or noise, which are called as *producer-to-consumption externality*.

In production externality the production of one firm is directly affected by the actions of another agent. For instance, the production of smoke by a steel mill may directly affect the production of clean clothes by a laundry, which is called as *producer-to-production externality*. The reduced contagion of disease due to vaccines may get healthy body and produce more output, which is called as *consumer-to-production externality*.

Recently, consumer-to-consumption externality has been extensively studied in the context of models of jealousy and "keeping up with the Joneses," such as Abel (1990) and Campbell and Cochrane (1999). Following Dupor and Liu (2003), we may say that the household feels either *jealous*, if marginal utility of aggregate consumption is negative, or *admiring*, if marginal utility of aggregate consumption is positive. In our paper, we employ aggregate consumption, C, in the utility function as consumer-to-consumption externality.

Producer-to-consumption externality has also been investigated in the context of models of public consumption and pollution such as Barro (1981), Huang and Cai (1994) and Chen, Lai and Shieh (2003). In Barro (1981), the household's utility is positively affected by public spending and if this spending is financed by income tax or factor taxation, the output or inputs affect externally the utility. Alternately, in Huang and Cai (1994) and Chen, Lai and Shieh (2003), the household's utility is negatively influenced by pollution from firms. Generally speaking, the pollution is positively related to firms' output or inputs and hence the utility is affected by output or inputs. Below, we choose output in the utility function to represent this externality.

Producer-to-production externality is frequently embedded in macroeconomic models. For example, in Romer (1986), the aggregate capital stock in the production function serves as a proxy for knowledge. Barro (1990) assumes that government expenditure is productive to analyze fiscal policies in an endogenous growth model. In these literatures, the output or inputs positively affect the production technology. Besides, the technology may be negatively affected by the output or inputs in a circumstance in which productive public spending is subject to some degree of congestion, such as Barro and Sala-i-Martin (1992), Glomm and Ravikumar (1994), Turnovsky (1996) and Chen and Lee (2007). Thus, we use production inputs to govern this externality in our paper.

Finally, the studies of consumer-to-production externality are not many in macroeconomic models. Kehoe, Levine and Romer (1991) and Drugeon (1998) tackle the issue of indeterminacy in an economy with aggregate consumption in the production technology. This setting can be motivated by the idea of empirical evidence in Leibenstein (1957) which indicates that nutrition improves productivity. As a result, the same setup is followed in our paper.

In the following sections, we will examine the relationship between externalities and efficiency in the long-run in some macroeconomic models.

3. Basic Model

3.1 Decentralized Economy

There is a representative agent with an infinite life whose labor supply is inelastic. It owns the shares of firms and decides the resource allocation toward consumption and investment at each instant of time *t*. There are four types of externality in this economy: (*i*) Consumer-to-consumption externality is represented by the aggregate consumption, *C*, in utility function; (*ii*) Producer-to-consumption externality is depicted by aggregate output, *Y*, in the utility function; (*iii*) Producer-to-production externality is described by aggregate capital, *K*, in Production function; (*iv*) Consumer-to-production externality is presented by the aggregate consumption in the production function. Taking externalities as given, the representative agent in the decentralized economy chooses individual consumption, *c*, and individual capital, *k*, to maximize the lifetime utility function

$$\int_0^{\infty} U(c,C,Y)e^{-\beta t}dt, \tag{1}$$

subject to the capital accumulation equation:

$$\dot{k} = F(k,K,C) - c - \delta k, \tag{2}$$

where $F(\cdot)$ is the individual production function with $y=F(k, K, C)$; $\beta>0$ is the rate of time preference and δ is the depreciation rate. We assume that the utility function is concave and the production function has the usual neoclassical properties, and both have sufficient curvature, the equilibrium conditions for this problem suffice to ensure that the interior optimum both exists and is unique. But the presence of externalities may cause potential problems, in that the concavity conditions that ensure a unique interior optimum may no longer hold. Throughout our analysis, we shall simply assume that the equilibrium conditions always yield a unique interior optimum.

Denote U_1, U_2 and U_3 as the marginal utility of private consumption, aggregate consumption and output, respectively; and F_1 and F_2 are the marginal product of private capital and aggregate capital, respectively. In aggregate, $c=C$ and $k=K$. The market equilibrium conditions are

$$U_1(C,C,Y)=\lambda, \tag{3a}$$

$$\lambda[F_1(K,K,C)-\delta]=\beta\lambda-\dot{\lambda}, \tag{3b}$$

$$\dot{K}=F(K,K,C)-C-\delta K, \tag{3c}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda K=0,$ where λ is the co-state variable associated with capital. Condition (3a) equates the marginal utility of consumption to shadow value of capital. Condition (3b) is the Euler equation for capital and Condition (3c) is the market-clearing condition.

3.2 Centrally Planned Economy

To derive the optimal allocation of the economy, we consider a social planner who, in maximizing the lifetime utility, (1), subject to the resource constraint $\dot{K}=F(K,K,C)-C-\delta K$, takes externalities into account.

Let starred variables denote the equilibrium of social optimum. The optimal equilibrium conditions are modified to

$$U_1(C^*,C^*,Y^*)+U_2(C^*,C^*,Y^*)+[U_3(C^*,C^*,Y^*)+\lambda^*]F_3(K^*,K^*,C^*)=\lambda^*, \tag{4a}$$

$$[U_3(C^*,C^*,Y^*)+\lambda^*][F_1(K^*,K^*,C^*)+F_2(K^*,K^*,C^*)]=(\beta+\delta)\lambda^*-\dot{\lambda}^*, \tag{4b}$$

$$\dot{K}^*=F(K^*,K^*,C^*)-C^*-\delta K^*, \tag{4c}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda^* K^*=0,$ where λ^* refers to the social co-state variable associated with capital. The structure of Conditions (4a)-(4c) parallels that of Conditions (3a)-(3c).

3.3 Comparison of steady-state equilibrium

We begin by comparing the steady-state equilibrium. For the decentralized economy, the steady-state equilibrium satisfies

$$U_1[C,C,F(K,K,C)]=\lambda, \tag{5a}$$

$$F_1(K,K,C)=\beta+\delta, \tag{5b}$$

$$F(K,K,C)-\delta K=C, \tag{5c}$$

The steady-state optimal conditions for the planner's problem are

$$U_1(C^*,C^*,Y^*)+U_2(C^*,C^*,Y^*)+[U_3(C^*,C^*,Y^*)+\lambda^*]F_3(K^*,K^*,C^*)=\lambda^*, \tag{6a}$$

$$[U_3(C^*,C^*,Y^*)+\lambda^*][F_1(K^*,K^*,C^*)+F_2(K^*,K^*,C^*)]=(\beta+\delta)\lambda^*, \tag{6b}$$

$$F(K^*,K^*,C^*)-\delta K^* = C^*, \tag{6c}$$

For the decentralized economy, (5b) and (5c) solve K, and C and hence λ is determined by (5a). Moreover, the social optimum λ^*, K^* and C^* are obtain by (6a)-(6c).

(a) Consumer-to-consumption Externality ($U_2 \neq 0$, $U_3=0$ and $F_2=F_3=0$)

Comparing (5b) and (5c) with (6b) and (6c), we find that $C=C^*$ and $K=K^*$ and thereby consumption externality have no effect on the steady-state equilibrium. So, this externality still implies efficiency.

(b) Producer-to-consumption Externality ($U_2=0$, $U_3 \neq 0$ and $F_2=F_3=0$)

Comparing (5b) and (5c) to (6b) and (6c), we find $C \neq C^*$ and $K \neq K^*$. Under $F_{11}+F_{12}<0$, $C<C^*$ and $K<K^*$ when $U_3>0$ and $C>C^*$ and $K>K^*$ when $U_3<0$. Thus, this externality distorts the long-run equilibrium to an inefficient allocation.

(c) Producer-to-Production Externality ($U_2=U_3=0$, $F_2 \neq 0$, and $F_3=0$)

Comparing (5b) and (5c) with (6b) and (6c), it is obvious that $C \neq C^*$ and $K \neq K^*$. We find that, under $F_{11}+F_{12}<0$ and $F_{21}+F_{22}<0$, $C<C^*$ and $K<K^*$ when $F_2>0$ and $C>C^*$ and $K>K^*$ when $F_2<0$. This implies that the long-run equilibrium is inefficient.

(d) Consumer-to-Production Externality ($U_2=U_3=0$, $F_2=0$, and $F_3 \neq 0$)

Similarly, this externality has no effect on the steady-state equilibrium. The result exerts efficient allocation.

According to above analysis, the statement that externalities are bound to generate inefficiency may be questioned. Then, we obtain the following proposition.

Proposition 1. *In a one-sector economy with inelastic labor supply, producer-to-consumption externality and producer-to-production externality indeed implies long-run inefficiency but consumer-to-consumption externality and consumer-to-production externality implies long-run efficiency.*

Here, consumer-generated externalities play no role in determining the steady state. This is because the determination of steady-state capital stock and consumption is related to the behavior of the choice of capital measured by Condition (6b) and market-clearing condition, (6c). Consumer-generated externalities which impact through the choice of consumption have no channel to affect the steady-state choice of capital and market-clearing condition.

4. Reexamining Producer-Generated Externalities

Do producer-generated externalities really imply inefficiency? In this section we continue to reexamine the relationship between producer-to-consumption and producer-to-production externalities and efficiency. We find that these externalities may exert an efficient allocation in some environments.

4.1 Producer-to-Consumption Externality

Assume that there are two sectors in this economy: one is the pure consumption sector (Sector y_1); the other is the pure investment sector (Sector y_2). Labor supply is also inelastic and total labor supply is normalized to be 1. The technologies for consumption goods and investment goods are

$$y_1 = F(k_1, l_1), \tag{7a}$$

$$y_2 = G(k - k_1, 1 - l_1), \tag{7b}$$

where k_1 and l_1 are capital and labor employed in the pure consumption sector and k is the total capital.

Assume that only the producer activities in Sector y_1 externally affect the utility of the representative agent. Thus, the lifetime utility function becomes

$$\int_0^\infty U(c, Y_1) e^{-\beta t} dt, \tag{8}$$

where Y_1 is the aggregate output in Sector y_1 which indicate the producer-to-consumption externality.

We consolidate the consumption and production sectors into a representative consumer-producer problem. Let the investment goods be the numeraire, and p be the price of consumption goods in terms of the investment goods. The representative agent's budget constraint is

$$\dot{k} = G(k - k_1, 1 - l_1) + pF(k_1, l_1) - pc - \delta k, \tag{9}$$

4.1.1 Decentralized Economy

Take externalities as given, the representative agent's problem is to choose c, k_1, l_1 and k in order to maximize the lifetime utility, (8), subject to (9). In aggregate, c=C, k=K, l_1=L_1 and k_1=K_1. The market equilibrium conditions are

$$U_1(C, Y_1) = p\lambda, \tag{10a}$$

$$pF_1(K_1, L_1) = G_1(K - K_1, 1 - L_1), \tag{10b}$$

$$pF_2(K_1, L_1) = G_2(K - K_1, 1 - L_1), \tag{10c}$$

$$G_1(K-K_1,1-L_1)-\delta-\beta=\frac{\dot{\lambda}}{\lambda}, \tag{10d}$$

$$C=F(K_1,L_1), \tag{10e}$$

$$\dot{K}=G(K-K_1,1-L_1)-\delta K, \tag{10f}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda K=0,$ where λ is the co-state variable associated with capital. Condition (10a) equates the marginal utility of consumption to the marginal cost that is the shadow price of the consumption good. Condition (10b) (*resp.* (10c)) requires the equalization of the value of the marginal products of capital (*resp.* of labor) in both sectors. While (10d) is the Euler equation for capital, (10e) and (10f) are the market-clearing conditions for pure consumption goods and pure investment goods, respectively.

4.1.2 Centrally Planned Economy

Taking externalities into account, the social planner is to maximize the lifetime utility with (8), subject to the resource constraint $C=F(K_1,L_1),$ and $\dot{K}=G(K-K_1,1-L_1)-\delta K.$ Then the optimal equilibrium conditions are

$$[U_1(C^*,Y_1^*)+U_2(C^*,Y_1^*)]F_1(K_1^*,L_1^*)=\lambda^* G_1(K^*-K_1^*,1-L_1^*), \tag{11a}$$

$$[U_1(C^*,Y_1^*)+U_2(C^*,Y_1^*)]F_2(K_1^*,L_1^*)=\lambda^* G_2(K^*-K_1^*,1-L_1^*), \tag{11b}$$

$$G_1(K^*-K_1^*,1-L_1^*)-\delta-\beta=\frac{\dot{\lambda}^*}{\lambda^*}, \tag{11c}$$

$$C^*=F(K_1^*,L_1^*), \tag{11d}$$

$$\dot{K}^*=G(K^*-K_1^*,1-L_1^*)-\delta K^*, \tag{11e}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda^* K^*=0,$ where λ^* is the *social* shadow value of capital.

4.1.3 Steady-State Comparisons

For the decentralized economy, dividing (10b) by (10c), the steady-state equilibrium satisfies

$$\frac{F_1(K_1,L_1)}{F_2(K_1,L_1)}=\frac{G_1(K-K_1,1-L_1)}{G_2(K-K_1,1-L_1)}, \tag{12a}$$

$$G_1(K-K_1,1-L_1)=\delta+\beta, \tag{12b}$$

$$C=F(K_1,L_1), \tag{12c}$$

$$G(K-K_1,1-L_1)=\delta K, \tag{12d}$$

This equilibrium system determines four endogenous variables: C, K, K_1, and L_1.

Dividing (11a) by (11b), the steady-state optimal conditions for the social planner's problem are

$$\frac{F_1(K_1^*,L_1^*)}{F_2(K_1^*,L_1^*)}=\frac{G_1(K^*-K_1^*,1-L_1^*)}{G_2(K^*-K_1^*,1-L_1^*)}, \tag{13a}$$

$$G_1(K^*-K_1^*,1-L_1^*)=\delta+\beta, \tag{13b}$$

$$C^*=F(K_1^*,L_1^*), \tag{13c}$$

$$G(K^*-K_1^*,1-L_1^*)=\delta K^*, \tag{13d}$$

It is easy to see that $C=C^*$, $K=K^*$, $L=L_1^*$ and $K_1=K_1^*$. Producer-to-consumption externality has no effect on steady-state equilibrium. In sum, we obtain

Proposition 2. *In a two-sector economy with inelastic labor supply, if the producer-to-consumption externality only comes from the production activities in the pure consumption sector, it still implies long-run efficiency.*

4.2 Producer-to-Production Externality

Assume that labor supply is also inelastic and there are two sectors in this economy: pure consumption sector (Sector y_1); pure investment sector (Sector y_2). Following Benhabib and Farmer (1996) and Benhabib and Nishimura (1998), the production externality is represented by sector-specific externality. Further, we assume that the sector-specific externality is only in Sector y_1. The technologies for consumption goods and investment goods are

$$y_1=F(k_1,l_1,K_1,L_1), \tag{14a}$$

$$y_2=G(k-k_1,1-l_1), \tag{14b}$$

Moreover, the lifetime utility function is

$$\int_0^\infty U(c)e^{-\beta t}dt, \tag{15}$$

and the representative agent's budget constraint is

$$\dot{k}=G(k-k_1,1-l_1)+pF(k_1,l_1,K_1,L_1)-pc-\delta k, \tag{16}$$

4.2.1 Decentralized Economy

Take externalities as given, the representative agent's problem is to choose c, k_1, l_1 and k in order to maximize the lifetime utility, (15), subject to (16). In aggregate, c=C, k=K, l_1=L_1 and k_1=K_1. The market equilibrium conditions are

$$U_1(C) = p\lambda, \tag{17a}$$

$$pF_1(K_1, L_1, K_1, L_1) = G_1(K - K_1, 1 - L_1), \tag{17b}$$

$$pF_2(K_1, L_1, K_1, L_1) = G_2(K - K_1, 1 - L_1), \tag{17c}$$

$$G_1(K - K_1, 1 - L_1) - \delta - \beta = \frac{\dot{\lambda}}{\lambda}, \tag{17d}$$

$$C = F(K_1, L_1, K_1, L_1), \tag{17e}$$

$$\dot{K} = G(K - K_1, 1 - L_1) - \delta K, \tag{17f}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda K = 0$, where λ is the co-state variable associated with capital. The meaning of these conditions is similar to that of (10a)-(10f).

4.2.2 Centrally Planned Economy

Taking externalities into account, the social planner is to maximize the lifetime utility, (15), subject to the resource constraints $C = F(K_1, L_1, K_1, L_1)$, and $\dot{K} = G(K - K_1, 1 - L_1) - \delta K.$ Then the optimal equilibrium conditions are

$$U_1(C^*)[F_1(K_1^*, L_1^*, K_1^*, L_1^*) + F_3(K_1^*, L_1^*, K_1^*, L_1^*)] = \lambda^* G_1(K^* - K_1^*, 1 - L_1^*), \tag{18a}$$

$$U_1(C^*)[F_2(K_1^*, L_1^*, K_1^*, L_1^*) + F_4(K_1^*, L_1^*, K_1^*, L_1^*)] = \lambda^* G_2(K^* - K_1^*, 1 - L_1^*), \tag{18b}$$

$$G_1(K^* - K_1^*, 1 - L_1^*) - \delta - \beta = \frac{\dot{\lambda}^*}{\lambda^*}, \tag{18c}$$

$$C^* = F(K_1^*, L_1^*, K_1^*, L_1^*), \tag{18d}$$

$$\dot{K}^* = G(K^* - K_1^*, 1 - L_1^*) - \delta K^*, \tag{18e}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda^* K^* = 0$, where λ^* is the *social* shadow value of capital.

4.2.3 Steady-State Comparisons

For the decentralized economy, dividing (17b) by (17c), the steady-state equilibrium satisfies

$$\frac{F_1(K_1, L_1, K_1, L_1)}{F_2(K_1, L_1, K_1, L_1)} = \frac{G_1(K - K_1, 1 - L_1)}{G_2(K - K_1, 1 - L_1)}, \tag{19a}$$

$$G_1(K - K_1, 1 - L_1) = \delta + \beta, \tag{19b}$$

$$C = F(K_1, L_1, K_1, L_1), \tag{19c}$$

$$G(K - K_1, 1 - L_1) = \delta K, \tag{19d}$$

Dividing (18a) by (18b), the steady-state optimal conditions for the social planner's problem are

$$\frac{F_1(K_1^*, L_1^*, K_1^*, L_1^*) + F_3(K_1^*, L_1^*, K_1^*, L_1^*)}{F_2(K_1^*, L_1^*, K_1^*, L_1^*) + F_4(K_1^*, L_1^*, K_1^*, L_1^*)} = \frac{G_1(K^* - K_1^*, 1 - L_1^*)}{G_2(K^* - K_1^*, 1 - L_1^*)}, \tag{20a}$$

$$G_1(K^* - K_1^*, 1 - L_1^*) = \delta + \beta, \tag{20b}$$

$$C^* = F(K_1^*, L_1^*, K_1^*, L_1^*), \tag{20c}$$

$$G(K^* - K_1^*, 1 - L_1^*) = \delta K^*. \tag{20d}$$

Now consider

Condition S1: $\frac{F_1}{F_2} = \frac{F_3}{F_4}$.

This condition means that marginal rate of technical substitution between capital and labor from the private perspective is equal to that from the social perspective. If production functions are multiplicative with their inputs, this condition implies that the capital intensity (or labor intensity) from the private perspective is identical to that from the social perspective. For example, let $y_1 = ak_1^{\alpha_1} l_1^{\alpha_2} K_1^{\psi_1} L_1^{\psi_2}$, where a is the productivity coefficient and ψ_1 and ψ_2 are the degree of externalities. Then, Condition S1 implies $\alpha_1/\alpha_2=(\alpha_1+\psi_1)/(\alpha_2+\psi_2)=\psi_1/\psi_2$. This condition is met in the following Cobb-Douglas production functions:

$Y_1 = aK_1^{\alpha_1} L_1^{\alpha_2} Y_1^{\phi}$;

$Y_1 = aK_1^{\alpha_1} L_1^{\alpha_2} T^{\phi}$, with government budget constraint, $T = \tau Y_1$, where T is the public spending and τ is the income tax rate.

Under Condition S1, $C=C^*$, $K=K^*$, $L_1=L_1^*$ and $K_1=K_1^*$ in the steady state. Therefore, we get the following proposition.

Proposition 3. *In a two-sector economy with inelastic labor supply, if the sector-specific externality is only in the pure consumption sector, producer-to-production externality still exerts long-fun efficiency under Condition S1.*

5. Elastic Labor Supply

If labor supply is elastic, consumer-generated externalities seems to exert inefficiency in the long-run, highlighted by Liu and Turnovsky (2005). Is it true under different environments? In this section, we take consumer-to-consumption externality an example to show that the answer is also no. We now consider a two-sector economy: pure consumption goods and consumable investment goods. The agent's lifetime preference and budget constraint are modified as follows.

$$\int_0^\infty U(c_1, c_2, 1-l_1-l_2, C_1, C_2)e^{-\beta t}dt, \tag{21}$$

$$\dot{k} = G(k-k_1, l_1) - c_2 + pF(k_1, l_2) - pc_1 - \delta k, \tag{22}$$

where c_1 and c_2 are the consumption of pure consumption goods and consumable investment goods, respectively; C_1 and C_2 are the corresponding aggregate consumption; l_1 and l_2 are the labor supply in the pure consumption goods sector and in the consumable investment goods sector, respectively.

5.1. Decentralized Economy

The representative consumer-producer's optimal program is to maximize (21) subject to (22). The equilibrium conditions, together with c_1=C_1 and c_2=C_2, l_1=L_1, l_2=L_2, k_1=K_1 and k=K are

$$U_1(C_1, C_2, 1-L_1-L_2, C_1, C_2) = p\lambda, \tag{23a}$$

$$U_2(C_1, C_2, 1-L_1-L_2, C_1, C_2) = \lambda, \tag{23b}$$

$$U_3(C_1, C_2, 1-L_1-L_2, C_1, C_2) = p\lambda F_2(K_1, L_1), \tag{23c}$$

$$U_3(C_1, C_2, 1-L_1-L_2, C_1, C_2) = \lambda G_2(K-K_1, L_2), \tag{23d}$$

$$pF_1(K_1, L_1) = G_1(K-K_1, L_2), \tag{23e}$$

$$G_1(K-K_1, L_2)\lambda = (\beta+\delta)\lambda - \dot{\lambda}, \tag{23f}$$

$$C_1 = F(K_1, L_1), \tag{23g}$$

$$\dot{K} = G(K-K_1, L_2) - C_2 - \delta K, \tag{23h}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda K = 0$, where λ is the co-state variable associated with capital. Conditions (23a) and (23b) mean that the marginal utility of the pure consumption goods and of the consumable investment goods is equal to their shadow price, respectively. Conditions (23c) and (23d) equate the marginal utility of leisure to the marginal utility obtained from the additional output if labor input is increased by one more unit in each sector. Other conditions are similar to (10b), (10d)-(10f).

5.2 Centrally Planned Economy

Taking externalities into account, the social planner is to maximize the lifetime utility (21), subject to the resource constraints $C_1 = F(K_1, L_1)$, and $\dot{K} = G(K - K_1, L_2) - C_2 - \delta K.$ Then the optimal equilibrium conditions are

$$U_2(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_5(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) = \lambda^*, \tag{24a}$$

$$\frac{U_3(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)}{U_1(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_4(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)} = F_2(K_1^*, L_1^*), \tag{24b}$$

$$U_3(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) = \lambda^* G_2(K^* - K_1^*, L_1^*), \tag{24c}$$

$$[U_1(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_4(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)]F_1(K_1^*, L_1^*) = \lambda^* G_1(K^* - K_1^*, L_1^*), \tag{24d}$$

$$G_1(K^* - K_1^*, L_1^*)\lambda^* = (\beta + \delta)\lambda^* - \dot{\lambda}^*, \tag{24e}$$

$$C_1^* = F(K_1^*, L_1^*), \tag{24f}$$

$$\dot{K}^* = G(K^* - K_1^*, L_1^*) - C_2^* - \delta K^*, \tag{24g}$$

with the transversality condition $\lim_{t\to\infty} e^{-\beta t}\lambda^* K^* = 0$. where λ^* is the social co-state variable associated with capital.

5.3 Steady-State Comparisons

For the decentralized economy, substituting (23a) and (23b) into (23c) and (23d) and after simplifying, the steady-state equilibrium satisfies

$$\frac{U_1(C_1, C_2, 1 - L_1 - L_2, C_1, C_2)}{U_2(C_1, C_2, 1 - L_1 - L_2, C_1, C_2)} = \frac{G_2(K - K_1, L_2)}{F_2(K_1, L_1)}, \tag{23c}$$

$$\frac{U_1(C_1, C_2, 1 - L_1 - L_2, C_1, C_2)}{U_2(C_1, C_2, 1 - L_1 - L_2, C_1, C_2)} = \frac{G_1(K - K_1, L_2)}{F_1(K_1, L_1)}, \tag{23d}$$

$$\frac{F_1(K_1, L_1)}{F_2(K_1, L_1)} = \frac{G_1(K - K_1, L_2)}{G_2(K - K_1, L_2)}, \tag{23e}$$

$$G_1(K - K_1, L_2) = (\beta + \delta), \tag{23f}$$

$$C_1 = F(K_1, L_1), \tag{23g}$$

$$G(K - K_1, L_2) = C_2 + \delta K, \tag{23h}$$

Substituting (24a) into (24c) and (24d) and after simplifying, the steady-state optimal conditions for the planner's problem are

$$\frac{U_1(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_4(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)}{U_2(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_5(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)} = \frac{G_2(K^* - K_1^*, L_2^*)}{F_2(K_1^*, L_1^*)}, \tag{23c}$$

$$\frac{U_1(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_4(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)}{U_2(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*) + U_5(C_1^*, C_2^*, 1 - L_1^* - L_2^*, C_1^*, C_2^*)} = \frac{G_1(K^* - K_1^*, L_2^*)}{F_1(K_1^*, L_1^*)}, \tag{23d}$$

$$\frac{F_1(K_1^*,L_2^*)}{F_2(K_1^*,L_2^*)}=\frac{G_1(K^*-K_1^*,L_2^*)}{G_2(K^*-K_1^*,L_2^*)}, \tag{23e}$$

$$G_1(K^*-K_1^*,L_1^*)=\beta+\delta, \tag{23f}$$

$$C_1^*=F(K_1^*,L_1^*), \tag{23g}$$

$$G(K^*-K_1^*,L_1^*)=C_2^*+\delta K^*. \tag{23h}$$

Consider

Condition S2: $\frac{U_1}{U_2}=\frac{U_4}{U_5}$.

This condition means that marginal rate of substitution between two consumption goods from the private perspective is equal to that from social perspective. This condition is met in the following functions:

$$U=\frac{[c_1C_1^{\psi}]^{1-\sigma_1}-1}{1-\sigma_1}+\frac{[c_2C_2^{\psi}]^{1-\sigma_2}-1}{1-\sigma_2}+\frac{[1-l_1-l_2]^{1-\sigma_3}-1}{1-\sigma_3},$$

$$U=\frac{[c_1C_1^{\psi}c_2C_2^{\psi}(1-l_1-l_2)^{\chi}]^{1-\sigma}-1}{1-\sigma},$$

$$U=\frac{[c_1C_1^{\psi}+\kappa_1c_2C_2^{\psi}+\kappa_2(1-l_1-l_2)^{\chi}]^{1-\sigma}-1}{1-\sigma},$$

$$U=\frac{[(c_1+C_1^{\psi})+\kappa_1(c_2+C_2^{\psi})+\kappa_2(1-l_1-l_2)^{\chi}]^{1-\sigma}-1}{1-\sigma},$$

$$U=\kappa c_1^{\phi_1}C_1^{\psi_1}c_2^{\phi_2}C_2^{\psi_2}(1-l_1-l_2)^{\chi},\ \text{with}\ \frac{\phi_1}{\phi_2}=\frac{\psi_1}{\psi_2}.$$

Under Condition S2, $C_1=C_1^*$, $C_2=C_2^*$, $L_1=L_1^*$, $L_2=L_2^*$, $K=K^*$, and $K_1=K_1^*$ in the steady state. Therefore, we get the following proposition.

Proposition 4. *In a two-sector economy with two consumption goods and elastic labor supply, consumer-to-consumption externality still implies long-run efficiency under Condition S2.*

This proposition indicates that externalities may lead to efficiency even if labor supply is elastic. The results depend on the environments.

6. Concluding Remarks（計畫成果自評）

This paper theoretically takes many examples to demonstrate that each kind of externalities may be Pareto irrelevant even if there are no transaction costs and coordinations across agents.

Different from the existing literature, we examine this relationship at steady state in a representative-agent macroeconomic model. This result is established when the distortions which made by externalities have no effect on the allocation of resources or are mutually canceled under some mild conditions or attainable environments. Thus, the classical argument that externalities lead to inefficiency may be questionable. In this respect, this result complements the assertion of Coase (1960), Buchanan and Stubblebine (1962) and Dahlman (1979).

Of course, these examples are not the end of the story. We can continue to do the same thing under different types of externalities and different environments. The reversed outcome may be obtained but the point is that externalities are unnecessary to imply inefficiency under some conditions. It indicates that policy intervention should be moderated in the economy with externalities even if you believe that the government handles externalities better than the market.

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