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污染防制、經濟成長與動態調整路徑之多重性

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計畫主持人：謝智源

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行政院國家科學委員會補助專題研究計畫 ☑ 成果報告 □期中進度報告

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中文摘要及關鍵詞(keywords)

本計畫設立一個簡單的內生成長模型來強調內生勞動供給與勞動在生產和防制污染部門間移動對於經濟成長現象影響的重要性。根據本研究的分析發現，當勞動休閒內生化決定時，生產外部性的存在性對於環境政策可以刺激經濟成長的共識將會受到挑戰。更明確地說，當勞動休閒內生化決定時，生產外部性是造成環境政策具有經濟成長效果的充分非必要條件。這一個結論和既存文獻，如 Bovenberg and Smulders (1996)與 Bovenberg and de Mooij (1997)等，是有很大的不同之處。更重要的是，在公部門防治污染與私部門的防治污染具有互補性質的假設前提之下，當私部門的防治污染支出愈多的經濟體系，公部門的污染防制行為對於經濟成長率刺激的效果也就愈明顯。

另外，本計畫同時分析經濟體系的短期調整行為。根據我們的分析發現，當公部門的污染支出愈大時，經濟體系產生多重調整路徑的機率也就愈高。也就是意味著公部門的防治污染支出規模更有可能是造成多重調整路徑的原因。

關鍵字：防治污染勞動投入、環境外部性、公共防治污染支出、雙重分紅、多重調整路徑

英文摘要及關鍵詞

This paper sets up a simple endogenous growth model that highlights the importance of the endogenous labor-leisure choice and the allocation between production labor and abatement labor. We show that, in contrast to the common notion (e.g., Bovenberg and Smulders, 1996 and Bovenberg and de Mooij, 1997), the existence of an environmental production externality is a sufficient (but not necessary) condition for environmental policies to stimulate economic growth if the labor-leisure choice is endogenously determined. In particular, since there are complementarities between public abatement and private abatement, the public abatement expenditure will have a more powerful enhancing effect on economic growth when it is accompanied by more efficient private abatement. This result also leads to a corollary to the effect that it is easier to achieve double dividends in terms of enhancing both growth and welfare if the endogenous labor-leisure choice is taken into account.

In our dynamic analysis, we show that if public abatement is substantially large, dynamic indeterminacy may occur despite the absence of a positive labor externality and, interestingly, this is more likely to be the case when abatement labor plays a more significant role. Besides, the transitional effects of an increase in public abatement are also investigated.

Keywords: Abatement labor; Environmental externality; Public abatement; Double dividends

JEL classification: O40, Q20

Growth, Welfare and Transitional Dynamics in an Endogenously Growing Economy with Abatement Labor

1. Introduction

Recently, there has been increased discussion concerning the impact of the environment on economic growth. Most existing studies relate environmental externalities to economic activities via households' utility, amenity and factor productivity. For example, Huang and Cai (1994), Ligthart and van der Ploeg (1994), Nielsen, *et al*. (1995) and Schou (2002) introduce environmental quality into their utility function to capture the amenity effect of a clean environment. Gradus and Smulders (1993), Bovenberg and Smulders (1995), Smulders and Gradus (1996), Mohtadi (1996), and Byrne (1997) by contrast emphasize the role of the environmental production externality in affecting economic growth. A common conclusion in these studies is that an ambitious environmental policy can stimulate economic growth as long as the environmental quality gives rise to a positive externality in regard to private production. Based on this argument, within endogenous growth frameworks Bovenberg and Smulders (1996) and Bovenberg and de Mooij (1997) further point out that environmental policies may yield double dividends by enhancing not only economic growth but also social welfare if the environmental production externality is sufficiently large.[1]

In departing from the above analyses, this paper uses a simple endogenous growth model to shed light on the importance of the endogenous labor-leisure choice and of the allocation between production and abatement labor that jointly govern the growth and welfare effects (and hence the possibility of double dividends) of environmental policies.[2] The inclusion of an endogenous

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[1] The double-dividend hypothesis can be broadly defined as follows: In addition to lowering the pollution level, environmental policies can achieve additional goals, such as lowering the unemployment rate, boosting the economy's growth rate, and increasing welfare. The relevant studies referred to include, for example, Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1998), Schneider (1997), Fisher and van Marrewijk (1998) and Strand (1998).

[2] It may be surprising that the role of the labor-leisure choice is virtually absent in the environmental literature. To our knowledge, Fisher and van Marrewijk (1998), Elbasha and Roe (1996), Byrne (1997), Hettich (1998) and Oueslati (2002) specify that labor can be allocated between different sectors, but is supplied inelastically.

allocation between production and abatement labor, on the one hand, reflects a realistic greater flexibility for firms to balance the tradeoffs between production and environmental degradation. On the other hand, it allows us to highlight the importance of interactions between private abatement and public abatement. With regard to the role of the labor-leisure choice, this has been emphasized in recent developments pertaining to endogenous growth theories (see, for example, Devereux and Love, 1995, Eriksson, 1996, Ladrón-de-Guevara, *et al.*, 1999, Turnovsky, 2000b, and Duranton, 2001).[3] In practice, labor is still an important productive input in the modern capitalistic economy. As noted by Mankiw (2000), the output elasticity with respect to labor is very high – about 0.7 in the U.S. between 1960 and 1996. Using U.S. time-series data from 1960 to 1990, Jones (1998) adopts Solow's (1957) "growth accounting equation" and shows that the GDP growth rate equals 3.1 percent per year, of which the labor growth rate accounts for 1.2 percent per year.

In this study we clearly point out that, even if the pollution externality in relation to prudcution is absent, public abatement can still stimulate growth provided that the labor-leisure choice is endogenously determined. Intuitively, to balance the government's budget constraint, public abatement expenditures must crowd out resources available to the private sector. This so-called *resources withdrawal effect* will reduce not only consumption but also leisure (if it is an endogenous variable) and, as a result, households will work more (both production and abatement labor will increase). Since labor and capital are technical complements under a Cobb-Douglas production technology, this will in turn encourage capital accumulation and speed up economic growth. It is important to note that, due to the fact that there are complementarities between public abatement and private abatement, the public abatement expenditure will have a more powerful enhancing effect on economic growth when it is accompanied by more efficient private abatement. These results lead to a corollary to the effect that the double dividends in terms of

[3] Turnovsky (2000a, p. 186) provide a timely comment in relation to endogenous growth models without an endogenous labor-leisure choice. He claims that "recent endogenous growth models have stressed the role of fiscal policy as a key determinant of long-run growth. One limitation of these new models is that with few exceptions they treat labor supply as inelastic, thereby abstracting from the decision to allocate time between work and leisure. This treatment severely limits certain aspects of fiscal policy, implying for example, that both a consumption tax and a tax on labor income operate as non-distortionary lump sum taxes."

improving both growth and welfare will be easier to achieve if the endogenous labor-leisure choice is taken into account.

In addition, this paper also engages in welfare and dynamic analyses. In the welfare analysis, we find that, given that public abatement and emission taxation are two possible instruments for a social planner to reach the Pareto optimum, public abatement is *not* able to serve as an instrument in remedying the environmental externality, in which case the optimal emission tax must seriously account for such externalities. To be more specific, the optimal public abatement increases as the extractive use of the natural environment becomes more productive. To eliminate externalities caused by pollution, the (modified) Pigouvian tax responds to the marginal damage not only to the households' utility but also to the firm's production. In the subsequent dynamic analysis, we show that if public abatement is substantially large, dynamic indeterminacy may occur despite the absence of a positive labor externality (that is commonly believed to be necessary for this transitional non-uniqueness, e.g., Benhabib and Farmer, 1994, and Benhabib and Perli, 1994). Interestingly, this is more likely to be the case when abatement labor plays a more significant role. Moreover, by focusing on the case of dynamic determinacy, we show that in response to an *anticipated* increase in public abatement, during the transition process the capital and consumption growth rates may exhibit a misadjustment from their steady state levels.

The remainder of this paper is organized as follows. Section 2 sets up an endogenous growth model highlighting the endogenous labor-leisure choice and the allocation between production and abatement labor. In Section 3 the existence and the dynamical stability of the competitive equilibrium are addressed. Section 4 uncovers the steady state effects of environmental policies and in turn the transitional effects are investigated in Section 5. In Section 6 we derive the first-best public abatement and emission tax. Finally, Section 7 concludes.

2. The model

Consider an economy that consists of a representative firm, a representative household and a government. There is only one homogeneous output, which is produced by the set of capital, labor, and the extractive use of the natural environment. The household derives utility from consumption, but incurs disutility from work and the damage caused by pollution. In order to manage the environment, the government considers emission taxation and more aggressively public abatement. To balance its budget, the government's abatement expenditures are financed by emission taxes and/or lump-sum taxes.

2.1. The firm

The representative firm produces output based on Cobb-Douglas technology as follows:

$$y = Ak^{\theta_1} e^{\theta_2} n_1^{\alpha} S^{-\beta}, \tag{1}$$

where *A* is a technology parameter, *k* is the physical capital, n_1 is the labor employed which is allocated to production, and *e* is the extractive use of the natural environment by the producer.[4] The parameters θ_1, θ_2 and α measure the weights of the private capital, the extractive use, and labor in relation to production, respectively. In order to ensure a positive but diminishing marginal productivity of these inputs, we assume that $0 < \theta_1, \theta_2, \alpha < 1$. In addition, by defining *S* as the aggregate pollution stock, $S^{-\beta}$ (with a positive parameter $\beta > 0$) captures the negative externality stemming from pollution damage.

To distinguish between gross and net pollution, we follow den Butter and Hofkes (1995) and Byrne (1997) and assume that the *net* pollutants discharged from firms amount to *p*. The net pollution increases with the *gross* emission of the firm (*e*), but decreases with the pollution abatement activity of the firm. To be specific, we assume that the firm allocates some proportion of labor n_2 to treat emissions and, accordingly, *p* is negatively related to n_2. Thus, we can specify that the net pollution takes the following form:

$$p = en_2^{-\upsilon}, \tag{2}$$

where v is the technology parameter of abatement labor. By referring to (1) and (2), the total labor *n* will be the sum of n_1 and n_2, i.e. $n = n_1 + n_2$.

4 As documented by Nielsen, *et al.* (1995, p. 188) "our treatment of pollution as an input reflects the idea that the services provided by the natural environment (including its function as a waste sink) enable the firm to increase its level of output for any given input of other factors."

There are perfectly competitive factor markets in which each firm faces a given interest rate r, a wage rate w and, in order to be permitted to emit pollution, pays emission tax to the government at the rate τ.[5] Given (1) and (2), the firm's optimization problem is to choose k, n_1, n_2 and e so as to maximize profits, π. That is:

$$\underset{k,n_1,n_2,e}{Max} \quad \pi = Ak^{\theta_1}e^{\theta_2}n_1^{\alpha}S^{-\beta} - w(n_1+n_2) - rk - \tau en_2^{-\upsilon}. \tag{3}$$

Equation (3) leads us to derive the first-order conditions as follows:

$$\theta_1 Ak^{\theta_1-1}e^{\theta_2}n_1^{\alpha}S^{-\beta} = r\,, \tag{4a}$$

$$\alpha Ak^{\theta_1}e^{\theta_2}n_1^{\alpha-1}S^{-\beta} = w\,, \tag{4b}$$

$$\upsilon\tau en_2^{-\upsilon-1} = w\,, \tag{4c}$$

$$\theta_2 Ak^{\theta_1}e^{\theta_2-1}n_1^{\alpha}S^{-\beta} = \tau n_2^{-\upsilon}. \tag{4d}$$

Equations (4a)-(4c) are the common $MR = MC$ conditions.

Substituting (4a)-(4d) into (3), the firm's profit function is given by:

$$\pi = (1-\alpha-\theta_1)y - (1+\upsilon)\tau p = [1-\alpha-\theta_1-\theta_2(1+\upsilon)]y\,. \tag{5}$$

Moreover, by putting (4b)-(4d) together, the firm's optimal allocation rule between production and abatement is given by:

$$n_2 = \frac{\upsilon\theta_2 n_1}{\alpha}. \tag{6}$$

This implies that, other things being equal, the firm will allocate more labor to engage in abatement activity the more productive that resource input e is (a higher θ_2) and the more skillful labor devoted to abatement n_2 is (a higher υ). However, the firm will replace abatement labor with production labor as labor devoted to production becomes more productive (a higher α).

2.2. The household

The objective of the representative household is to maximize the discounted sum of future instantaneous utilities. That is:

$$Max \quad \int_0^{\infty}[\Lambda_1 \ln c - \Lambda_2\frac{n^{1+\varepsilon}}{1+\varepsilon} - \Lambda_3\frac{S^{1+\psi}}{1+\psi}]e^{-\rho t}dt\,; \quad \Lambda_1,\Lambda_2,\Lambda_3 > 0\,, \tag{7}$$

[5] For the sake of simplification, the wage rate for production and abatement labor is assumed to be identical. This assumption will not qualitatively alter our main results.

where ρ is the subjective time preference rate. The parameters ε and ψ measure the impact of work and pollution on the household's satisfaction, respectively. In order to satisfy the requirement that work and the pollution stock yield negative and diminishing marginal utility, we impose the restrictions $\varepsilon > 0$ and $\psi > 0$.[6]

The representative household is bound by a flow constraint linking capital accumulation to any difference between its disposable incomes (wage income wn, capital income $rk + \pi$ and lump-sum transfers (or tax) $tr > 0$ ($tr < 0$)) and consumption expenditure. Thus, the household budget constraint can be described as:

$$\dot{k} = wn + rk + \pi - c + tr , \tag{8}$$

where an overdot denotes the rate of change with respect to time. Following the common assumption in the relevant literature, such as in Ligthart and van der Ploeg (1994), Michel and Rotillon (1995), Elbasha and Roe (1996), and Bovenberg and de Mooij (1997), the household treats environmental pollution as given since the household feels that its activities are too insignificant to affect the overall pollution level. With this assumption, the household chooses consumption and work to maximize the discounted sum of utility defined in (7), subject to the budget constraint (8).

The optimal conditions necessary for this optimization problem are given by:

$$\frac{\Lambda_1}{c} = \lambda , \tag{9a}$$

$$\Lambda_2 n^{\varepsilon} = w\lambda , \tag{9b}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r , \tag{9c}$$

together with the budget constraint (8) and the transversality condition $\lim_{t\to\infty} \lambda k e^{-\rho t} = 0$. The term λ is the co-state variable, which can be interpreted as the shadow value of the capital stock, measured in terms of utility. Equation (9a) indicates that the co-state variable λ is equal to the marginal utility of consumption. Equation (9b) reports that the marginal disutility of work is equal to the marginal benefit from work. The differential equation (9c) is the Euler equation for physical capital, indicating that the change in the shadow value of capital depends on the

[6] See Keeler, *et al.* (1972) and Tahvonen and Kuuluvainen (1991) for detailed discussions.

difference between the rate of time preference and the rental rate.

Totally differentiating (9a) with respect to time and substituting (9c) into the resulting equation yields the optimal intertemporal consumption rule:

$$\dot{c} = (r - \rho)c\,. \tag{10}$$

Equation (10) is the well-known Keynes-Ramsey rule.

2.3. Ecological system

In line with the common specification in the environmental literature, the pollution stock grows as the net emission *p* increases, and declines as the government's public abatement *M* increases. Thus, given a constant natural decay rate of pollution δ, the pollution stock accumulates in the following manner:

$$\dot{S} = \frac{p}{M} - \delta S\,. \tag{11}$$

2.4. The government

Devereux and Love (1995, p. 236) claim that "government spending must persistently rise if government is to remain a significant fraction of the economy." Turnovsky (2000a, p. 433) similarly points out that "[i]n order to sustain an equilibrium with steady growth, government expenditure cannot be fixed at some exogenous level, as it has been previously, but rather must be linked to the scale of the economy in some way." In line with their argument and to sustain a continual growth rate, we assume that the government sets its public abatement expenditure as a fixed fraction of output, that is:

$$M = \phi y\,,\quad 0 < \phi < 1\,, \tag{12a}$$

where ϕ is the public abatement expenditure share.

The government collects its emission tax revenue to finance its expenditure on pollution abatement and transfers. Thus, the government's budget constraint can be expressed as:

$$M + tr = \tau p\,. \tag{12b}$$

To isolate the growth effect of environmental policies, we further assume that the government balances its budget at any instant in time by adjusting the lump-sum term *tr*.

In addition, by substituting the government's budget constraint (12b) with the functional form of public abatement expenditure (12a), the producers' profits (5), and the optimization condition for producers (4a)-(4d) into the household's budget constraint (8), the economy-wide resource constraint is given by:

$$\dot{k} = (1-\phi)Ak^{\theta_1}e^{\theta_2}n_1^{\alpha}S^{-\beta} - c\,. \tag{13}$$

3. Competitive dynamic equilibrium

The competitive dynamic equilibrium is defined as a set of market clearing prices and quantities such that:

i. the firm maximizes profits, i.e. (4a)-(4d);

ii. the representative household maximizes its lifetime utility, i.e. (8) and (9a)-(9c);

iii. the government budget constraint is balanced, i.e. (12a) and (12b);

iv. the law of motion for the pollution stock is satisfied, i.e. (11).

A non-degenerate balanced-growth path (BGP) is a tuple of paths $\{c,k,e,y,w,r,n_1,n_2\}_{t=0}^{\infty}$ such that each of the quantity variables *c*, *k*, *e* and *y* (hence *M*) grows at a positive constant rate and the price variables *w* and *r* as well as working hours n_1 and n_2 (hence *n*) are positively constant. Furthermore, in accordance with Smulders and Gradus (1996), Bovenberg and Smulders (1996), Elbasha and Roe (1996), and Bovenberg and de Mooij (1997), in equilibrium the growth rate of the pollution stock must be zero to avoid the over degeneration of the environmental quality, i.e. $(\dot{S}/S)^* = 0$ is satisfied along the BGP (hereafter, the superscript "*" denotes the steady-state value for relevant variables). In order to ensure a constant rate of economic growth along a BGP, here we should assume that $\theta_1 + \theta_2 = 1$. To simplify our notation, we further specify that $\theta_1 = \theta$ and $\theta_2 = 1-\theta$.

It is easily seen from (4a), (10), (12a) and (13) that in BGP equilibrium the economy exhibits *common growth* in which consumption, capital, emissions, output, and public abatement expenditure all grow at a common rate γ^*, i.e.:

$$\gamma^* = (\dot{c}/c)^* = (\dot{k}/k)^* = (\dot{e}/e)^* = (\dot{y}/y)^* = (\dot{M}/M)^*.$$

We are now ready to solve the dynamic system. By substituting (4b)-(4d) into the labor market clearing condition $n = n_1 + n_2$, we first have the following relationship:

$$n_1 = \varsigma n, \tag{14a}$$

$$n_2 = (1-\varsigma) n, \tag{14b}$$

where $\varsigma = \alpha / [\alpha + \upsilon(1-\theta)]$. Equations (14a) and (14b) imply that the firm allocates a constant fraction of total labor hours to production and allocates the remaining fraction to treat emissions. Moreover, by putting (4d), (6) and (14a) together, we obtain:

$$\frac{e}{k} = [\frac{(1-\theta)\Phi A n^{\alpha+\upsilon} S^{-\beta}}{\tau}]^{1/\theta}, \tag{15}$$

where $\Phi = [\upsilon(1-\theta)]^{\upsilon} \alpha^{\alpha} / [\alpha + \upsilon(1-\theta)]^{\alpha+\upsilon} > 0$. We then define the transformed variable: $x \equiv c/k$ and, accordingly, (4b), (9a), (9b) and (14a) can be consolidated to remove the co-state variable λ, yielding:

$$\Lambda_2 n^{\varepsilon} = \Lambda_1 \alpha A (\frac{e}{k})^{1-\theta} (\varsigma n)^{\alpha-1} S^{-\beta} x^{-1}. \tag{16}$$

Equation (16) is in essence the optimal consumption-leisure condition in which the *MRS* in terms of consumption and leisure (labor supply) equals the relative price ratio.

By substituting (15) into (16), we can remove e/k and then derive the instantaneous relationship of total working hours as follows:

$$n = n(x, S, \tau); \; n_x = -\Omega\theta x^{-1} n, \; n_s = -\Omega\beta n S^{-1}, \; n_{\tau} = -(1-\theta)\Omega n \tau^{-1}, \tag{17}$$

where $\Omega = 1/[\theta(1+\varepsilon) - \upsilon(1-\theta) - \alpha] > 0$.[7] Based on (17), the instantaneous relationship of n_1 and n_2 can easily be obtained.

By manipulating (4a), (10), (13), (14a) and (15) with the *n* function (17), we have:

$$\frac{\dot{x}}{x} = \Gamma (n^{\alpha+\upsilon(1-\theta)} S^{-\beta})^{1/\theta} + x - \rho, \tag{18}$$

where $\Gamma = (\phi - 1 + \theta)\varsigma^{\alpha} [(1-\theta)\Phi \tau^{-1}]^{(1-\theta)/\theta} A^{1/\theta}$. Furthermore, by substituting (1), (2), (4d) and (12a) into (11), the evolution of the pollution stock is represented as:

$$\dot{S} = \frac{1-\theta}{\tau\phi} - \delta S. \tag{19}$$

Differential equations (19) and (18) together with the instantaneous relationship (17) thus

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[7] We assume that the marginal cost of labor (in terms of utility) is increasing and the marginal benefit of labor (in terms of utility) is decreasing. Given this assumption, the restriction $\theta(1+\varepsilon) > \alpha + \upsilon(1-\theta)$ (i.e. $\Omega > 0$ in (17)) holds.

constitute the 2×2 dynamic system in terms of the transformed variable x and the pollution stock S, leading to:

***Theorem 1.* (*Existence of the Non-degenerate BGP*) *There exists a unique balanced growth equilibrium in the economy if public abatement is relatively small* (*i.e.* $\phi < 1-\theta$). *However, the economy is characterized by two equilibria if public abatement is substantially large* (*i.e.* $\phi > 1-\theta$).**

Proof: The non-degenerate BGP equilibrium is characterized by $\dot{x}=0$ in (18) and $\dot{S}=0$ in (19). First of all, from (19) with $\dot{S}=0$, we can derive the pollution stock in equilibrium as: $S^* = (1-\theta)/(\tau\phi\delta)$. By substituting this result into (17) and (18) with $\dot{x}=0$, we have the following relationship in terms of the steady state x^*:

$$x^* = G(x^*) = \rho - \Gamma(n^{*\alpha+\upsilon(1-\theta)} S^{*-\beta})^{1/\theta}. \tag{20}$$

As shown in Figure 1, the function $G(x^*) = \rho - \Gamma(n^{*\alpha+\upsilon(1-\theta)} S^{*-\beta})^{1/\theta}$ may be either monotonically increasing and concave or decreasing and concave in x^*, crucially depending on $\phi \gtrless 1-\theta$, i.e.

$$G'(x^*) = -\frac{\Gamma[\alpha+\upsilon(1-\theta)](n^{*\alpha+\upsilon(1-\theta)-\theta} S^{*-\beta})^{1/\theta} n_x}{\theta} \gtrless 0 \quad if \quad \phi \gtrless 1-\theta, \tag{21a}$$

$$G''(x) = -\frac{\theta(1+\varepsilon)[\alpha+\upsilon(1-\theta)]\Gamma\Omega^2(n^{*\alpha+\upsilon(1-\theta)} S^{*-\beta})^{1/\theta}}{x^{*2}} \gtrless 0 \quad if \quad \phi \lessgtr 1-\theta, \tag{21b}$$

where $n_x = -\Omega\theta x^{-1} n < 0$ derived from (17). In addition, from (16) and (20), $\lim_{x\to 0} G(x^*) = -\infty$ if $\phi > 1-\theta$ (as shown in Figure 1a) and $\lim_{x\to 0} G(x^*) = \infty$ if $\phi < 1-\theta$ (as shown in Figure 1b). Thus, by applying the fixed point theorem, if $G(x^*)$ is upward sloping the dynamic system may be characterized by two equilibria (as shown in Figure 1a), while if the $G(x^*)$ locus is downward sloping a stationary x^* exists and is unique.[8] ■

We now turn to the investigation of dynamic stability. Linearizing (18) and (19) around the BGP values, namely, x^* and S^*, yields:

$$\begin{bmatrix} \dot{x} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x - x^* \\ S - S^* \end{bmatrix} + \begin{bmatrix} a_{13}d\phi + a_{14}d\tau \\ a_{23}d\phi + a_{24}d\tau \end{bmatrix}. \tag{22}$$

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[8] If $\phi = 1-\theta$, the $G(x^*)$ function is reduced to a horizontal line. In such a case, the equilibrium exists and is also unique.

The exact expressions of a_{ij} for $i=1,2$ and $j=1,\cdots,4$ are arranged in Appendix A. Accordingly, we have:

Proposition 1. (*Dynamic Determinacy and Indeterminacy*) *Given Theorem 1, the equilibrium of the economy is locally determinate if:*

$$\rho < \Theta = \frac{\theta(1+\varepsilon)x^*}{\alpha + \upsilon(1-\theta)} .$$

Otherwise, local indeterminacy occurs under the (necessary) condition $\phi > 1-\theta$ ***(public abatement is substantially large). Of importance, abatement labor will increase the likelihood of local indeterminacy.***

Proof: By defining μ_1 and μ_2 as the two characteristic roots of the dynamic system, from (22) we have:

$$\mu_1 = a_{11} = x^* + \Omega[\alpha + \upsilon(1-\theta)](x^* - \rho) \gtrless 0 , \tag{23a}$$

$$\mu_2 = a_{22} = -\delta < 0 , \tag{23b}$$

$$\mu_1\mu_2 = \Delta = -\delta\{x^* + \Omega[\alpha + \upsilon(1-\theta)](x^* - \rho)\} \gtrless 0 . \tag{23c}$$

As addressed in the literature on perfect-foresight models, such as Burmeister (1980) and Buiter (1984), the dynamic system will have a unique perfect-foresight equilibrium path if the number of (positively) unstable roots equals the number of jump variables. Given that $\mu_2 < 0$ and there is one jump variable x in this system, the equilibrium will be locally determinate if the dynamic system is characterized by saddle-path stability (i.e. one root with a negative real part and one root with a positive real part) in which $\Delta < 0$ holds. By contrast, there exists a continuum of equilibrium trajectories that converges to the steady state and, accordingly, local indeterminacy emerges in the economy if the dynamic system is a sink (i.e. two roots with negative real parts).

To be more specific, in our model the dynamic system displays dynamic determinacy (indeterminacy) if $\mu_1 > 0$ ($\mu_1 < 0$), or equivalently:

$$\rho < (>)\, \Theta = \frac{\theta(1+\varepsilon)x^*}{\alpha + \upsilon(1-\theta)} .$$

The above condition reveals that to have dynamic indeterminacy in this dynamic environmental model, the following two conditions: (i) the necessary (but not sufficient) condition $\phi > 1-\theta$ (hence $\Theta > 0$) and (ii) the necessary and sufficient condition $\rho < \Theta$ must be satisfied. Furthermore, it is found that $\partial\Theta / \partial\upsilon < 0$, implying that the likelihood of local indeterminacy increases as abatement labor becomes more effective (υ becomes larger). ■

Proposition 1 points out that the dynamic system is locally determinate if public abatement is *not* substantially large such that $\phi \leq 1-\theta$ (hence $\Theta \leq 0$) is true. However, if public abatement is substantially large ($\phi > 1-\theta$ and hence $\Theta > 0$), dynamic indeterminacy may occur provided that the condition $\rho < \Theta$ is met. Of importance, when abatement labor plays a more significant role (υ is larger) local indeterminacy is more likely to become true. This gives an implication that dynamic indeterminacy may occur despite the absence of a *positive labor externality* that is commonly believed to be necessary for this transitional non-uniqueness (see Benhabib and Famer, 1994, and Benhabib and Perli, 1994).

Figure 2 is helpful to our understanding of the dynamic stability in this system. From (22), it is easy to derive:

$$\left.\frac{dS}{dx}\right|_{\dot{x}=0} = -\frac{a_{11}}{a_{12}} = -\frac{\{x^* + \Omega[\alpha + \upsilon(1-\theta)](x^* - \rho)\}S^*}{\beta(1+\varepsilon)\Omega x^*(x^* - \rho)} \gtrless 0 \quad \text{and} \quad \left.\frac{dS}{dx}\right|_{\dot{S}=0} = -\frac{a_{21}}{a_{22}} = 0 .$$

Given that $x^* - \rho = -(\phi - 1 + \theta)\varsigma^{\alpha}\{[(1-\theta)\Phi\tau^{-1}]^{1-\theta} An^{*\alpha+\upsilon(1-\theta)} S^{*-\beta}\}^{1/\theta}$, this indicates that the $\dot{S} = 0$ locus is horizontal and the $\dot{x} = 0$ curve may be either upward or downward sloping. Obviously, if the dynamic system is locally indeterminate, Proposition 1 indicates that $\phi > 1-\theta$ (hence $x^* - \rho < 0$) and $\rho > \Theta$ (hence $a_{11} < 0$) and, accordingly, the slope of $\dot{x} = 0$ is negative, as shown in Figure 2. By contrast, if the dynamic system is locally determinate, the condition $a_{11} > 0$ must hold in order to ensure saddle point stability. As a result, Figure 2 demonstrates that the slope of $\dot{x} = 0$ is crucially dependent upon $\phi \gtrless 1-\theta$.[9] Thus, by referring to Figure 2, we learn that the equilibrium point E_3 (in which the government's abatement expenditure is relatively large) is characterized by local indeterminacy and E_0, E_1 and E_2 (in which the government's abatement expenditure is relatively low) are characterized by local determinacy.

[9] Note that the $\dot{x} = 0$ locus reduces to a vertical line if $\phi = 1-\theta$. In such a case, the steady state consumption-capital rate turns out to be $x^* = \rho$.

In addition, Figure 2 also reveals that if $\phi > 1-\theta$, there exist two equilibria in the economy, as indicated in Theorem 1.

4. The effects of environmental policies

In this section we will investigate the effects of the public abatement share ϕ and emission taxation τ on the pollution stock, working hours (production labor and abatement labor), the growth rate, and welfare. For our purposes, the analysis will focus on the case of local determinacy, i.e. $\Delta < 0$.

4.1. The environmental policy effects on pollution, employment, and economic growth

From (13), (14a) and (15) we have:

$$\gamma^* = (\frac{\dot{k}}{k})^* = \frac{(1-\phi)\Gamma(n^{*\alpha+\upsilon(1-\theta)}S^{*-\beta})^{1/\theta}}{\phi-1+\theta} - x^* . \tag{24}$$

Based on (22) and (24) with $\dot{x} = \dot{S} = 0$, we can establish the following proposition:

Proposition 2. (*The Pollution, Employment, and Growth Effects*) *Given Theorem 1, in the case of local determinacy we have:*

$$\frac{dS^*}{d\phi} < 0 ; \ \frac{dn^*}{d\phi} > 0 ; \ \frac{d\gamma^*}{d\phi} > 0 , \tag{25}$$

$$\frac{dS^*}{d\tau} < 0 ; \ \frac{dn^*}{d\tau} \gtrless 0 ; \ \frac{d\gamma^*}{d\tau} \gtrless 0 , \ if \ 1-\theta \lessgtr \beta . \tag{26}$$

Proof: See Appendix B.

Proposition 2 first points out that different environmental policies – both public abatement and emission taxation – can improve the quality of the environment. Given that these two policies alleviate the negative externality caused by pollution, Proposition 2 further indicates that, while an increase in public abatement expenditure stimulates the steady-state growth rate, increasing emission tax has an ambiguous effect on the balanced-growth rate due to its additional distortion effect.

Generally speaking, the impact of public abatement expenditure on the growth rate is the joint operating consequence of both *the resources withdrawal effect* and *the pollution externality effect*. The resources withdrawal effect indicates that, given the government's budget constraint, more public abatement expenditure is associated with fewer transfers, and hence with the amount of resources available to the private sector. When the household's income is reduced, not only will consumption decrease but leisure will also decrease and, as a result, the household will work more, i.e. $dn^* / d\phi > 0$. Since an increase in the marginal product of private capital is associated with an increase in labor supply (*n* and *k* are technical complements), this will in turn encourage capital accumulation and speed up economic growth.

The pollution externality effect indicates that an increase in public abatement expenditure will reduce pollution damage. Given that the unfavorable production externality is alleviated, the balanced-growth rate rises in response. Since both the resources withdrawal and pollution externality effects give rise to a positive effect on capital accumulation, the result $d\gamma^* / d\phi > 0$ is true.

In addition to the pollution externality effect of public abatement, emission taxation gives rise to an additional *distortionary tax effect* in terms of affecting the input factor's price and, in turn, employment and growth.[10] More specifically, a higher emission tax will result in emission input *e* becoming more expensive, thereby discouraging the firm from demanding emission input. Since the Cobb-Douglas production reported in (1) implies that *e* and *n* are technical complements, the demand for labor decreases as well. Once the working hours fall, the marginal productivity of capital decreases and economic growth follows. Given that this effect is the opposite of the pollution externality effect, emission taxation will have an ambiguous effect on employment and growth.

One point should be emphasized. If the labor supply is exogenously determined (i.e. $\alpha = \varepsilon = \upsilon = 0$) and the pollution externality in relation to production is absent ($\beta = 0$), public abatement will have no impact on the economic growth rate (i.e. $d\gamma^* / d\phi\big|_{\substack{\beta=0 \\ n=\bar{n}}} = 0$). This result is consistent with that of Ligthart and van der Ploeg (1994). When we take the pollution externality in relation to production into account ($\beta > 0$), leaving the labor supply exogenously

[10] Note that emission taxation, on the one hand, decreases the household's income (due to the decrease in the firm's profits), and, on the other hand, increases its transfer income (due to the balance on the government's budget), having no impact on the household's income. Therefore, emission taxation does not generate any resources withdrawal effect on economic growth.

fixed, the growth effect of public abatement is reduced to $(d\gamma^* / d\phi)\big|_{n=\bar{n}} > 0$. This result confirms the argument in relevant studies, such as Gradus and Smulders (1993), Bovenberg and Smulders (1995), Smulders and Gradus (1996), Mohtadi (1996), and Byrne (1997). Finally, when we isolate the effect of the pollution externality ($\beta = 0$) and emphasize the role of the labor-leisure choice, the growth effect will turn out to be $(d\gamma^* / d\phi)\big|_{\beta=0} > 0$. This result contributes to an important implication in that, to reap a positive growth effect for the environmental policy, the existence of a pollution externality in relation to production is *not* a necessary condition.

Abatement labor also plays a role, affecting the growth effect of the environmental policy. The comparative statics of Proposition 2 allow us to have:

Corollary 1. ***(The Intensifying Effect of the Abatement Labor) The growth effect of the public abatement is reinforced by a more effective private abatement (a higher υ).***

Proof: From (25), we immediately have:

$$\frac{\partial^2 \gamma^*}{\partial \phi \partial \upsilon} = -\frac{\theta(1-\theta)(1+\varepsilon)x^*(x^* - \rho)}{\phi(\phi - 1 + \theta)\{\theta(1+\varepsilon)x^* - \rho[\alpha + \upsilon(1-\theta)]\}^2}\{\rho\beta + \frac{\theta\phi(\rho - x^*)}{\phi - 1 + \theta}\} > 0. \blacksquare$$

Corollary 1 implicitly reveals that there are complementarities between public abatement and private abatement; the public abatement expenditure ϕ has a more powerful enhancing effect on economic growth when it is accompanied by more efficient private abatement than when it is not.

4.2. The welfare effect and double dividends

In endogenous growth models, Bovenberg and Smulders (1996) and Bovenberg and de Mooij (1997) emphasize that, if the environmental production externality is sufficiently large, the environmental policies may yield double dividends by enhancing both economic growth and social welfare. In departing from their argument, we will show in what follows that, if labor supply is endogenously determined, the environmental policies (with a particular emphasis on the pubic abatement effect) may reap such a twofold dividend despite the absence of a pollution externality in relation to production (i.e. $\beta = 0$).

In line with Greiner and Hanusch (1998), we only deal with the welfare effect in the BGP equilibrium. Along the BGP, given the initial capital stock k_0 and consumption c_0, both consumption and the capital stock grow at a common rate γ^* (which is a function of ϕ and τ), but the growth rates of both the pollution stock and working time are zero, as noted previously.

Welfare, denoted by W, is the utility obtained by the representative household, as reported in (7), i.e. $W = \int_0^\infty [\Lambda_1 \ln c - \Lambda_2 \frac{n^{1+\varepsilon}}{1+\varepsilon} - \Lambda_3 \frac{S^{1+\psi}}{1+\psi}] e^{-\rho t} dt$. Given the balanced-growth rate γ^*, the time path of consumption can be expressed as:

$$c_t = c_0 e^{\gamma^* t}, \tag{27}$$

Using (4a), (10), (13) and (15), we can solve c_0 which is related to k_0:

$$c_0 = \frac{(1-\theta-\phi)\gamma^* + \rho(1-\phi)}{\theta} k_0 . \tag{28}$$

Substituting (19), (27), and (28) into the welfare function above and differentiating the resulting equation with respect to ϕ, we have the following proposition:

Proposition 3. (*The Welfare Effect*) *Given Theorem 1, in the case of local determinacy the welfare effect of public abatement is given by:*

$$\frac{\partial W}{\partial \phi} = -[\frac{\Lambda_1 k_0}{\rho c_0} \frac{\rho + \gamma^*}{\theta}] - [\frac{\Lambda_2 n^{*\varepsilon}}{\rho} \frac{\partial n^*}{\partial \phi}] + [\frac{\Lambda_3 S^{*1+\psi}}{\rho \phi}] + [\frac{(1-\theta-\phi)\Lambda_1 k_0}{\rho \theta c_0} + \frac{\Lambda_1}{\rho^2}] \frac{\partial \gamma^*}{\partial \phi} \lesseqgtr 0, \tag{29}$$

Equation (29) clearly indicates that the effect of public abatement on social welfare consists of four distinctive components that may counteract each other. The first term on the RHS in (29) is "*the resource mobilization effect*," which indicates that an increase in public abatement expenditure crowds out private consumption (hence decreases the consumption-capital ratio) and, consequently, decreases welfare. The second term simply captures "*the labor disutility effect*," which indicates that welfare decreases as the disutility from working increases when public abatement induces households to work more. The third term is "*the environmental amenity value effect*," which shows that public abatement can improve social welfare through a reduction in the damage caused by pollution. Finally, the fourth term is "*the economic growth effect*," which captures the impact of public abatement on the balanced-growth rate. As indicated in Proposition 2, the fourth effect crucially depends on whether the labor-leisure variable is endogenous and on whether the pollution externality in relation to production is present.[11]

Based on Propositions 2 and 3, we immediately have:

Corollary 2. (***Double Dividends***) ***If the labor supply is endogenously determined, an increase in pubic abatement may result in a twofold dividend in terms of improving both economic growth and social welfare despite the absence of a pollution externality in relation to production.***

Corollary 2 tells us that when the pollution externality in relation to production ($\beta = 0$) is absent and if n is endogenously determined, an increase in public abatement, on the one hand, unambiguously increases the balanced-growth rate and, on the other hand, is more likely to improve social welfare *via* the increase in the economic growth effect under the condition of local determinacy $1-\theta-\phi>0$.[12] Therefore, the so-called double dividends are easier to achieve if the endogenous labor-leisure choice is taken into account.

5. Transitional dynamics

In practice, the environmental authorities usually implement their policies with a pre-announcement. There is a typical example: on July 13, 2000, the U.K. Treasury announced that the government budget for the Environmental Protection Agency would increase by 15% every year over the next three years. A similar situation also occurred in Japan, where in 2004 the Environment Ministry also claimed that the environmental tax on fossil fuels would be increased over the next 2-3 years in order to deal with global warming. To incorporate this observation into our model, in this section we will deal with an anticipated change in public abatement.

By using a graphical apparatus such as that in Figure 2, under the equilibrium of the saddle-point stability this section proceeds to trace the possible transitional patterns in terms of a shock as public abatement expenditure occurs. This analysis can easily be extended to investigate the effects of changing the emission tax. However, because this issue has been

[11] Following a similar approach, the welfare effect of a tax emission is easy to determine. The mathematical deduction is available upon request.

[12] The restriction $1-\theta-\phi>0$ is the sufficient condition for local determinacy.

debated extensively in the environmental literature, we will abstract it from our analysis. Moreover, given that the dynamics are qualitatively similar regardless of the slope of the $\dot{x}=0$ locus being negative or positive, for the sake of brevity, we will report the case where the $\dot{x}=0$ locus is downward sloping (i.e. the $\phi<1-\theta$ case) only.

In Figure 3 we consider an economy initially at a BGP equilibrium with a saddle point, E_0, corresponding to the initial consumption-capital ratio and pollution of x_0 and S_0, respectively. At time $t=0$, the authority announces that the share of public abatement expenditure will permanently rise from ϕ_0 to ϕ_1 at $t=T$ in the future. In response to such an anticipated increase in public abatement, the $\dot{S}=0(\phi_0)$ locus will shift downward to $\dot{S}=0(\phi_1)$, while $\dot{x}=0(\phi_0)$ will shift leftward to $\dot{x}=0(\phi_1)$.[13] As a result, Figure 3a (Figure 3b) indicates that the new steady-state value of the consumption-capital ratio x^* is less (greater) than x_0 and the new steady-state value of the pollution stock S^* is less than S_0. These results are confirmed in (25) and (26).

Before proceeding to study the economy's dynamic adjustment, three points should be addressed. First, for expository convenience, in what follows 0^- and 0^+ denote the instant before and after the policy announcement, respectively, while T^- and T^+ denote the instant before and after the policy's implementation, respectively. Second, during the dates between 0^+ and T^-, public abatement expenditure remains at its initial level ϕ_0, and point E_0 should be treated as the reference point that governs the dynamic adjustment of *x* and *S*. However, since the public knows that the public abatement share will increase from ϕ_0 to ϕ_1 at the moment T^+, the transversality condition requires that the economy move to a point on the convergent stable branch associated with ϕ_1, $SS(\phi_1)$, at that instant of time. Third, since *x* is a jump variable it will respond immediately to the shock. However, due to the fact that *S* is predetermined at the instant 0^+, it must be fixed at S_0 at that moment.

With this background, the transitional path is exhibited by either the trajectory $E_0E_{0+}E_TE^*$

[13] Under the condition $\phi<1-\theta$, (22) leads to: $\partial S/\partial\phi|_{\dot{x}=0}=-a_{13}/a_{12}<0$ and $\partial S/\partial\phi|_{\dot{S}=0}=-a_{23}/a_{22}<0$.

(shown in Figures 3a) or $E_0 E'_{0+} E'_T E^*$ (shown in Figure 3b), this depending crucially on whether the productivity of private abatement labor is high enough or not ($\upsilon > \hat{\upsilon} = \frac{\theta(1+\varepsilon)-\alpha}{1-\theta} + \frac{\beta(1+\varepsilon)(\phi-1+\theta)}{\phi(1-\theta)}$).[14] Given these two distinctive paths, the transitional behaviors of the rates of capital and consumption growth are also obtained easily. By applying Barro and Sala-i-Martin's (2004, pp. 257-262) approach, we manipulate the formulas for $\dot{k}/k$ ($=\gamma_k$) (from (13), (15) and (17)) and $\dot{c}/c$ ($=\gamma_c$) (from (4a), (10), (14a), (15) and (16)), and use these formulas to sketch the transitional dynamics of γ_k and γ_c, as shown in Figures 4a and 4b.[15] In sum, we have:

Proposition 3. (*Transitional Paths*) *In response to an anticipated rise in public abatement, the transitional dynamics exhibit the following properties:*

(i) ***If the productivity of abatement labor is relatively low ($\nu < \hat{\upsilon}$), once the policy is announced, the consumption growth rate monotonically increases until the balanced-growth rate is reached. The capital growth rate increases during the period following the policy announcement, but prior to the policy's implementation. At the instant of the policy's implementation, the capital growth rate decreases discretely. Afterwards, it continues to rise to the balanced-growth rate;***

(ii) ***If the productivity of abatement labor is relatively low ($\nu > \hat{\upsilon}$), the short-run growth rates of capital and consumption exhibit a "misadjustment" from the steady state level: they fall during the period between the policy announcement and its implementation, but they increase to the balanced-growth rate when the policy is actually implemented.***

6. The first-best environmental policies

Due to the presence of pollution externalities, the market equilibrium in our model is inefficient. In the Pareto optimum, the social planner will internalize the pollution externalities

[14] From footnote 13, the critical value of $\hat{\upsilon}$ is immediately derived.

[15] A detailed deduction is available upon request.

in order to remedy this inefficiency. By comparing the comparative equilibrium with the Pareto optimum, in this section we will explore the first-best environmental policies ϕ and τ.

The social planner, subject to the aggregate resource constraint (13), the evolution of pollution (30), and the market-clearing condition for labor (31), maximizes the social welfare function by choosing *c*, *e*, n_1, n_2, *k* and *S*. That is:

$$Max \quad \int_0^\infty [\Lambda_1 \ln c - \Lambda_2 \frac{n^{1+\varepsilon}}{1+\varepsilon} - \Lambda_3 \frac{S^{1+\psi}}{1+\psi}] e^{-\rho t} dt; \qquad \Lambda_1, \Lambda_2, \Lambda_3 > 0, \tag{7}$$

$$\text{s.t.} \quad \dot{k} = (1-\phi) A k^\theta e^{1-\theta} n_1^\alpha S^{-\beta} - c. \tag{13}$$

$$\dot{S} = \frac{e n_2^{-\upsilon}}{\phi A k^\theta e^{1-\theta} n_1^\alpha S^{-\beta}} - \delta S, \tag{30}$$

$$n = n_1 + n_2. \tag{31}$$

By letting μ and η be the co-state variables associated with the aggregate resource constraint and the law of motion for pollution, the optimal conditions for this optimization problem are given by:

$$\frac{\Lambda_1}{c} = \mu, \tag{32a}$$

$$(1-\phi)(1-\theta) A k^\theta e^{-\theta} n_1^\alpha S^{-\beta} \mu = -\frac{\theta n_2^{-\upsilon} \eta}{\phi A k^\theta e^{1-\theta} n_1^\alpha S^{-\beta}}, \tag{32b}$$

$$\Lambda_2 (n_1 + n_2)^\varepsilon = (1-\phi) \alpha A k^\theta e^{1-\theta} n_1^{\alpha-1} S^{-\beta} \mu - \frac{\alpha n_2^{-\upsilon} \eta}{\phi A k^\theta e^{-\theta} n_1^{1+\alpha} S^{-\beta}}, \tag{32c}$$

$$\Lambda_2 (n_1 + n_2)^\varepsilon = -\frac{\upsilon n_2^{-\upsilon-1} \eta}{\phi A k^\theta e^{-\theta} n_1^\alpha S^{-\beta}}, \tag{32d}$$

$$\frac{\dot{\mu}}{\mu} = \rho - (1-\phi) \theta A k^{\theta-1} e^{1-\theta} n_1^\alpha S^{-\beta} + \frac{\theta n_2^{-\upsilon} \eta}{\phi A k^{1+\theta} e^{-\theta} n_1^\alpha S^{-\beta} \mu}, \tag{32e}$$

$$\frac{\dot{\eta}}{\eta} = \rho + \frac{\Lambda_3 S^\psi}{\eta} + \frac{(1-\phi) \beta A k^\theta e^{1-\theta} n_1^\alpha S^{-\beta-1} \mu}{\eta} - \frac{\beta n_2^{-\upsilon}}{\phi A k^\theta e^{-\theta} n_1^\alpha S^{1-\beta}} + \delta. \tag{32f}$$

Using the labor market-clearing condition (31), and (32b)-(32d), we have:

$$n_1 = \frac{\alpha}{\alpha + \upsilon(1-\theta)} n = \varsigma n, \tag{33a}$$

$$n_2 = \frac{\upsilon(1-\theta)}{\alpha + \upsilon(1-\theta)} n = (1-\varsigma) n. \tag{33b}$$

It follows from (14a), (14b), (33a), and (33b) that there are identical n_1 and n_2 in the competitive equilibrium and the Pareto optimum, implying that via the market mechanism the allocation for labor is Pareto optimal.

For our purposes, we only focus on the social optimum in the BGP equilibrium. By letting superscript "o" be the first-best tax rate associated with the relevant variables, we can thus establish:

Proposition 4. (*The First-Best Environmental Policies*) *In the presence of environmental externalities, the first-best public abatement and emission tax are given by, respectively:*

$$\phi^o = 1-\theta > 0 . \tag{34}$$

$$\tau^o = \frac{1}{\Xi(1-\theta)y} MRS + \frac{\theta\beta}{\Xi(1-\theta)S} > 0 , \tag{35}$$

where $\Xi = [\rho - \frac{\beta n_2^{-\upsilon}}{\phi A k^{\theta} e^{-\theta} n_1^{\alpha} S^{1-\beta}} + \delta] > 0$ and $MRS = -(\Lambda_3 S^{\psi})/(\Lambda_1 / c) > 0$.

Proof: By comparing (9a) with (32a), we learn that $\lambda = \mu$. As a result, utilizing (4a), (9c), (32b) and (32e) immediately leads to:

$$\phi^o = 1-\theta .$$

In addition, from (4c), (32b) and (34), we have:

$$\tau^o = -\frac{\theta\eta}{\phi(1-\phi)y\mu} .$$

Given $\phi^o = 1-\theta$ and substituting (32f) into the above equation with $\dot{\eta} = 0$, the first-best emission tax can be represented as:

$$\tau^o = \frac{1}{\Xi(1-\theta)y} MRS + \frac{\theta\beta}{\Xi(1-\theta)S} . \blacksquare$$

Proposition 4 reveals that, to reach the Pareto optimum, the government should provide positive abatement expenditure. In the face of a more productive emission input (i.e. $(1-\theta)$ is higher), firms will be induced to emit more pollution, and therefore public abatement must increase in order to optimally control the pollution stock. Although public abatement is not used to remedy the distortion caused by environmental externalities, a positive emission tax will account for such externalities. Given that $\phi^o = 1-\theta$, in the absence of a pollution externality arising from production of ($\beta = 0$), the first-best emission tax reported in (35) is reduced to $\tau^o = MRS / [\Xi(1-\theta)y]$, which is essentially the Pigouvian tax. This implies that the socially-optimal emission tax must respond to the marginal damage in order to eliminate the effect of the pollution externality on the households' utility. If the pollution externality arising from production is taken into account, (35) indicates that the optimal rate of the emission tax will increase when pollution gives rise to a highly unfavorable effect on the production of the good (β is larger).

Besides, it may also be interesting to note that, in the Pareto optimum, public abatement and emission taxation are substitutes. By substituting (34) into (35), we have:

$$\tau^o = \frac{1}{\Xi \phi^o y} MRS + \frac{(1-\phi^o)\beta}{\Xi \phi^o S} \quad \text{and} \quad \frac{\partial \tau^o}{\partial \phi^o} < 0,$$

which confirms our argument.

7. Concluding Remarks

In this paper we have shown that the environmental production externality and the endogenous labor supply (and the allocation between production and abatement labor) are two equally important elements governing the steady-state and dynamic effects of environmental policies on employment, growth and welfare. First of all, our model indicates that if public abatement is substantially large, dynamic indeterminacy may occur despite the absence of a positive labor externality. It is interesting to note that this is more likely to be the case when abatement labor plays a more significant role. Second, in contrast to the common notion that the existence of an environmental production externality is necessary for environmental policies to boost economic growth (e.g., Bovenberg and Smulders, 1996 and Bovenberg and de Mooij, 1997), this paper has clearly pointed out that, even without the pollution externality arising from production, public abatement can also stimulate growth provided that the labor-leisure choice is endogenously determined. Since there are complementarities between public abatement and private abatement, the public abatement expenditure will have a more powerful enhancing effect on economic growth when it is accompanied by more efficient private abatement. This result also leads to a corollary to the effect that the double dividends in terms of improving both growth and welfare are easier to achieve if the endogenous labor-leisure choice is taken into account.

Third, to reach the Pareto optimum, the government should provide positive abatement expenditures that increase when the emission input is more productive. While public abatement is not used to remedy the distortion caused by environmental externalities, a positive emission tax will account for such externalities. This modified Pigouvian tax, on the one hand, responds to the marginal damage in order to eliminate the effect of the pollution externality on the households' utility. On the other hand, it must increase if pollution gives rise to a more unfavorable effect on the production of the good.

Appendix A

Linearizing (18) and (19) around the steady-state equilibrium and substituting the steady-state relationship $x^* - \rho = \Gamma(n^{*\alpha+\upsilon(1-\theta)} S^{*-\beta})^{1/\theta}$ into the resulting equations, the exact derivations of a_{ij} for $i = 1,2$ and $j = 1,\cdots,6$ are expressed as follows:

$$a_{11} = x^* + \Omega[\alpha + \upsilon(1-\theta)](x^* - \rho) \gtrless 0,$$

$$a_{12} = \frac{\beta(1+\varepsilon)\Omega x^*(x^* - \rho)}{S^*} \gtrless 0 \quad if \quad \phi \lessgtr 1-\theta,$$

$$a_{13} = x^*(\rho - x^*)/(\phi - 1 + \theta) > 0,$$

$$a_{14} = \frac{(1-\theta)(1+\varepsilon)\Omega x^*(x^* - \rho)}{\tau} \gtrless 0 \quad if \quad \phi \lessgtr 1-\theta,$$

$$a_{21} = 0,$$

$$a_{22} = -\delta < 0,$$

$$a_{23} = -\frac{\delta S^*}{\phi} < 0,$$

$$a_{24} = -\frac{\delta S^*}{\tau} < 0.$$

Appendix B

Using (22) with $\dot{x} = \dot{S} = 0$, we obtain the following comparative statics:

$$\frac{dx^*}{d\phi} = \frac{\delta[\phi + \beta(1+\varepsilon)(\phi - 1 + \theta)\Omega]x^*(\rho - x^*)}{\phi(\phi - 1 + \theta)\Delta} \gtrless 0, \qquad \frac{dS^*}{d\phi} = -\frac{S^*}{\phi} < 0,$$

$$\frac{dx^*}{d\tau} = \frac{\delta(1+\varepsilon)(1-\theta-\beta)\Omega x^*(x^* - \rho)}{\tau\Delta} \gtrless 0, \qquad \frac{dS^*}{d\tau} = -\frac{S^*}{\tau} < 0.$$

By combining the above comparative statics with (13), (15), and (17), the effects of ϕ and τ on the balanced-growth rate and labor supply are given by:

$$\frac{d\gamma^*}{d\phi} = \frac{\theta\delta\Omega(x^* - \rho)}{\phi(\phi - 1 + \theta)\Delta}\{\beta(1+\varepsilon)x^* + \frac{\phi[\alpha + \upsilon(1-\theta)](\rho - x^*)}{\phi - 1 + \theta}\} > 0,$$

$$\frac{dn^*}{d\phi} = -\frac{\delta\Omega n^*}{\phi\Delta}\{\rho\beta + \frac{\theta\phi(\rho - x^*)}{\phi - 1 + \theta}\} > 0,$$

$$\frac{d\gamma^*}{d\tau} = \frac{\theta\delta(1-\theta-\beta)(1+\varepsilon)\Omega x^*(\rho - x^*)}{\tau(\phi - 1 + \theta)\Delta} \gtrless 0 \quad if \quad 1-\theta \lessgtr \beta,$$

$$\frac{dn^*}{d\tau} = \frac{\delta\rho(1-\theta-\beta)\Omega n^*}{\tau\Delta} \gtrless 0 \quad if \quad 1-\theta \lessgtr \beta.$$

References

Benhabib, J. and R.E.A. Farmer, 1994, Indeterminacy and increasing returns, Journal of Economic Theory 63, 19-41.

Benhabib, J. and R. Perli, 1994, Uniqueness and indeterminacy: On the dynamics of endogenous growth, Journal of Economic Theory 63, 113-142.

Bovenberg, A. L. and F. van der Ploeg, 1998, Tax reform, structural unemployment and the environment, Scandinavian Journal of Economics 100, 593-610.

Bovenberg, A. L. and R. A. de Mooij, 1994, Environmental levies and distortionary taxtation, American Economic Review 84, 1085-1089.

Bovenberg, A. L. and R. A. de Mooij, 1997, Environmental tax reform and endogenous growth, Journal of Public Economics 63, 207-237.

Bovenberg, A. L. and S. Smulders, 1995, Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model, Journal of Public Economics 57, 369-391.

Bovenberg, A. L. and S. Smulders, 1996, Transitional impacts of environmental policy in an endogenous growth model, International Economic Review 37, 861-893.

Buiter, W. H., 1984, Saddlepoint problems in continuous time rational expectations models: A general method and some macroeconomic examples, Econometrica 52, 665-680.

Burmeister, E., 1980, On some conceptual issues in rational expectations modeling, Journal of Money, Credit, and Banking 12, 800-812.

Byrne, M. M., 1997, Is growth a dirty word? Pollution, abatement and economic growth, Journal of Development Economics 54, 261-284.

Devereux, M. B. and D. R. F. Love, 1995, The dynamic effects of government spending policies in a two-sector endogenous growth model, Journal of Money, Credit, and Banking 27, 232-256.

den Butter, F. A. G. and M. W. Hofkes, 1995, Sustainable development with extractive and non-extractive use of the environment in production, Environmental and Resource Economics 6, 341-358.

Duranton, G., 2001, Endogenous labor supply, growth and overlapping generations, Journal of Economic Behavior and Organization 44, 295-314.

Elbasha, E. H. and T. L. Roe, 1996, On endogenous growth: The implications of environmental externalities, Journal of Environmental Economics and Management 31, 240-268.

Eriksson, C., 1996, Economic growth with endogenous labour supply, European Journal of Political Economy 12, 533-544.

Fisher, E. O. and C. van Marrewijk, 1998, Pollution and economic growth, Journal of International Trade and Economic Development 7, 55-69.

Gradus, R. and S. Smulders, 1993, The trade-off between environmental care and long-term growth: Pollution in three prototype growth models, Journal of Economics 58, 25-51.

Greiner, M. and H. Hanusch, 1998, Growth and welfare effects of fiscal policy in an endogenous growth model with public investment, International Tax and Public Finance 5, 249-261.

Hettich, F, 1998, Growth effects of a revenue-neutral environmental tax reform, Journal of Economics 67, 287-316.

Huang, C. H. and D. Cai, 1994, Constant returns endogenous growth with pollution control, Environmental and Resource Economics 4, 383-400.

Keeler, E., M. Spence, and R. Zeckhauser, 1972, The optimal control of pollution, Journal of Economic Theory 4, 19-34.

Ladrón-de-Guevara, A., S. Ortigueira, and S. Santos, 1999, A two-sector model of endogenous growth with leisure, Review of Economic Studies 66, 609-631.

Ligthart, J. E. and F. van der Ploeg, 1994, Pollution, the cost of public funds and endogenous growth, Economic Letters 46, 351-361.

Michel, P. E. and G. Rotillon, 1995, Disutility of pollution and endogenous growth, Environmental and Resource Economics 6, 279-300.

Mohtadi, H., 1996, Environment, growth, and optimal policy design, Journal of Public Economics 63, 119-140.

Nielsen, S. B., L. H. Pedersen, and P. B. Sørensen, 1995, Environmental policy, pollution, unemployment, and endogenous growth, International Tax and Public Finance 2, 183-204.

Oueslati, W, 2002, Environmental policy in an endogenous growth model with human capital and endogenous labor supply, Economic Modeling 19, 487-507.

Schneider, K., 1997, Involuntary unemployment and environmental policy: The double dividend hypothesis, Scandinavian Journal of Economics 99, 45-59.

Schou, P., 2002, Pollution externalities in a model of endogenous fertility and growth, International Tax and Public Finance 9, 709-725.

Smulders, S. and R. Gradus, 1996, Pollution abatement and long-term growth, European Journal of Political Economy 12, 505-532.

Strand, J., 1998, Pollution taxation and revenue recycling under monopoly unions, Scandinavian Journal of Economics 100, 765-780.

Tahvonen, O. and J. Kuuluvainen, 1991, Optimal growth with renewable resources and pollution, European Economic Review 35, 650-661.

Turnovsky, S. J., 2000a, Methods of Macroeconomic Dynamics, 2nd Edition. Cambridge, MA: The MIT Press.

Turnovsky, S. J., 2000b, Fiscal policy, elastic labor supply, and endogenous growth, Journal of Monetary Economics 45, 185-210.

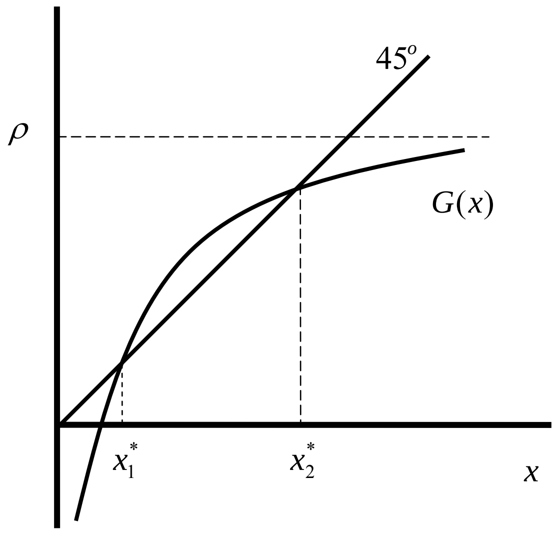

Figure 1a. The $\phi > 1-\theta$ case

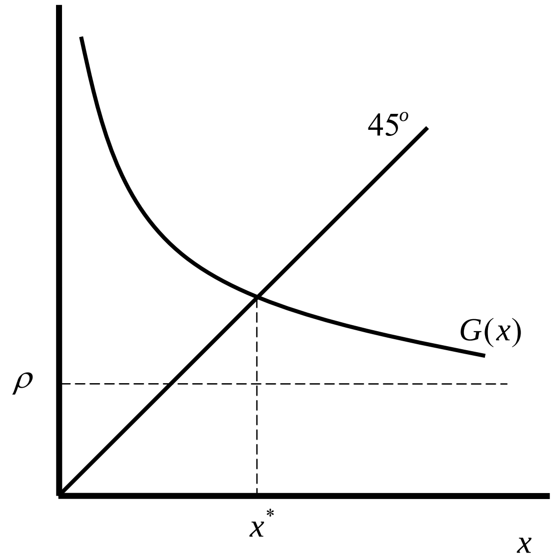

Figure 1b. The $\phi < 1-\theta$ case

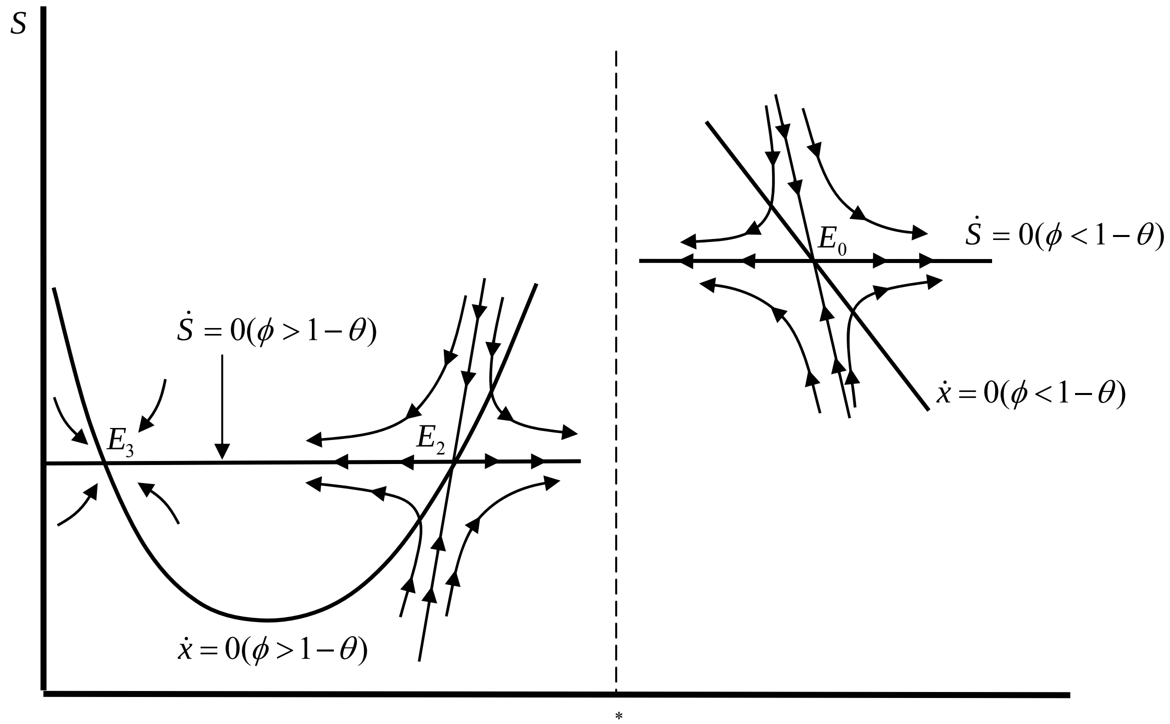

Figure 2. Phase diagram

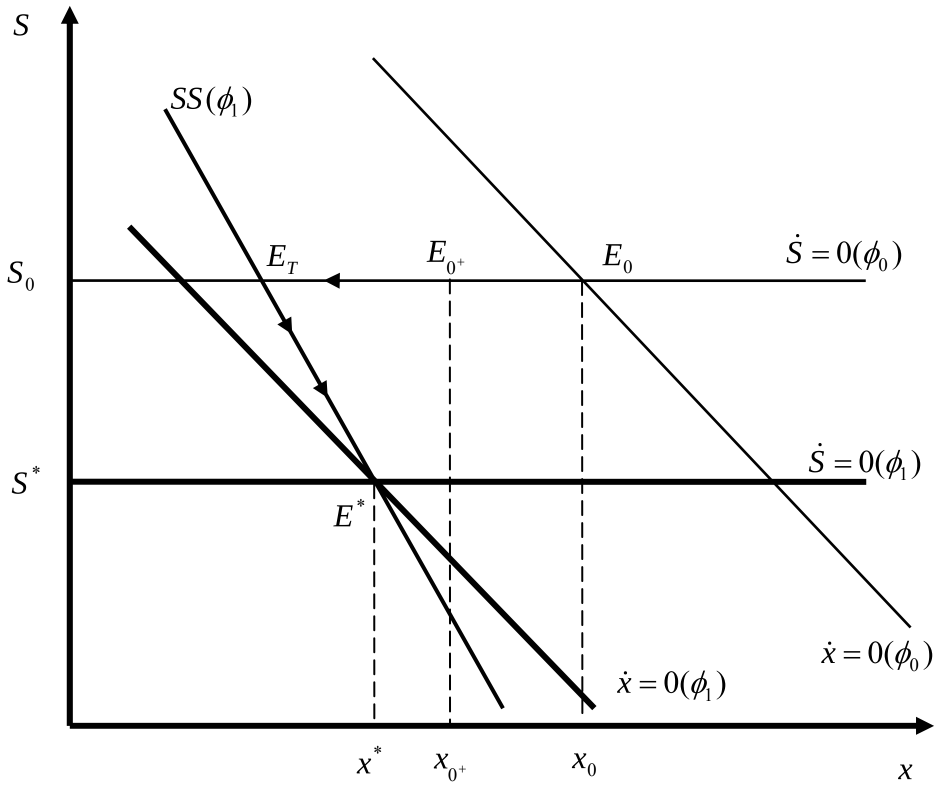

Figure 3a Transitional dynamics: The $\phi < 1-\theta$ case with $\upsilon < \hat{\upsilon}$.

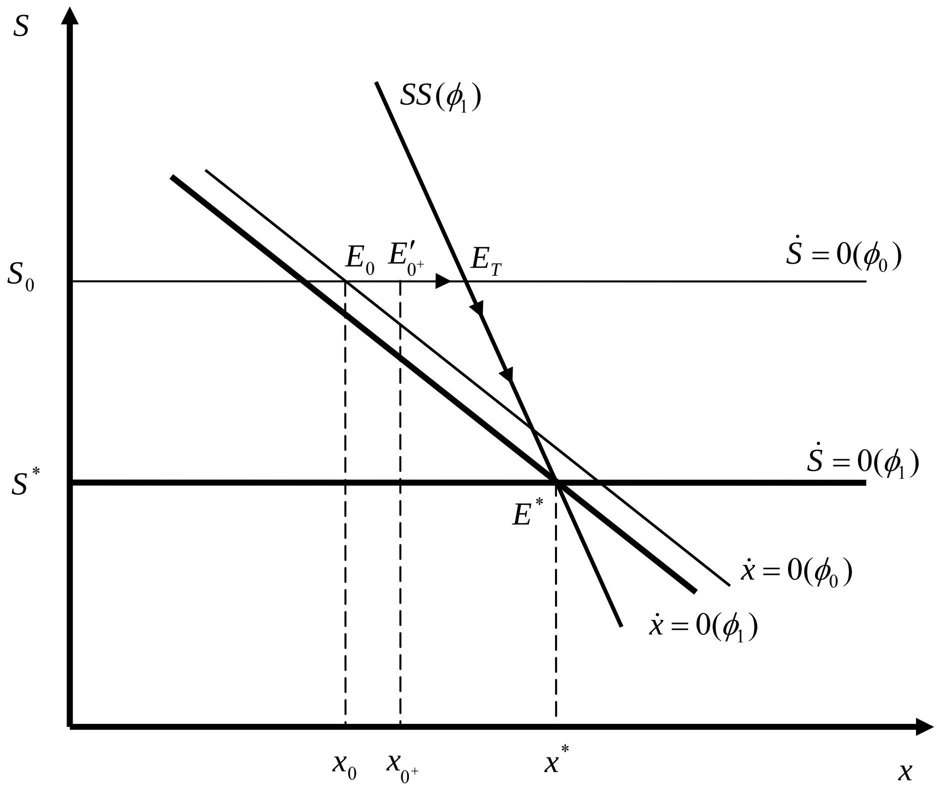

Figure 3b Transitional dynamics: The $\phi < 1-\theta$ case with $\upsilon > \hat{\upsilon}$.

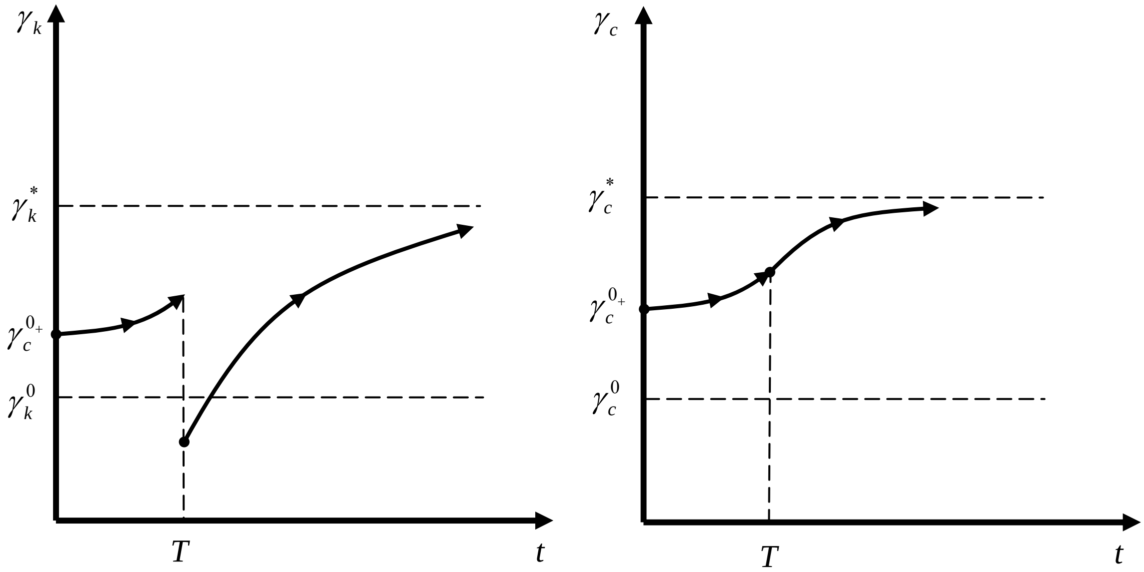

Figure 4a Dynamic adjustment of γ_k and γ_c: the $\phi < 1-\theta$ case with $\upsilon < \hat{\upsilon}$.

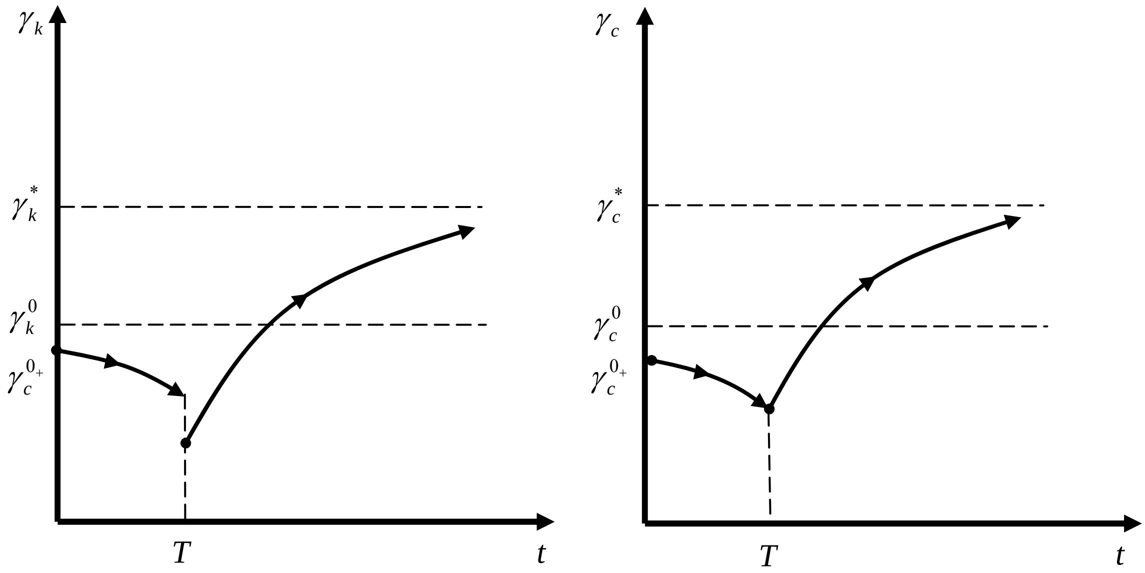

Figure 4a Dynamic adjustment of γ_k and γ_c: the $\phi < 1-\theta$ case with $\upsilon > \hat{\upsilon}$.

附件二

可供推廣之研發成果資料表

□ 可申請專利　　□ 可技術移轉　　日期：95 年 08 月 10 日

國科會補助計畫	計畫名稱：**污染防制、經濟成長與動態調整路徑之多重性** 計畫主持人：**謝智源** 計畫編號：**NSC 94－2415－H－034－003－** 學門領域：**農業與自然資源經濟學**
技術/創作名稱	
發明人/創作人	
技術說明	中文： （100~500 字）
	英文：
可利用之產業及可開發之產品	
技術特點	
推廣及運用的價值	

※　1.每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送　貴單位研發成果推廣單位（如技術移轉中心）。

※　**2.本項研發成果若尚未申請專利，請勿揭露可申請專利之主要內容。**

※　3.本表若不敷使用，請自行影印使用。