中國文化大學 105 學年度碩士班考試入學招生考試試題

系所組：化學工程與材料工程學系奈米材料碩士班　節次：第 1 節

科目：工程數學

1. (50%) Solve y(x) for the following differential equations.
   (a) $y''-6'+9y=0$ with $y(0)=2$ and $y'(0)=8$ (15%)
   (b) $y''+4y'+3y=\sin x+2\cos x$ with $y(0)=0$ and $y'(0)=1.5$ (20%)
   (c) $(1-x^2)\,dy+4xydx=0$ (15%)
2. (10%) Derive the Laplace transform for the function {cosh kt} is

   $\mathcal{L}\{\cosh kt\}=\frac{s}{s^2-k^2}$

3. (15%) matrix $M=\begin{pmatrix} 1 & \sqrt{8} & 0 \\ \sqrt{8} & 1 & \sqrt{8} \\ 0 & \sqrt{8} & 1 \end{pmatrix}$, Please find the three eigenvalues of the matrix M.
4. (25%) Using the Fourier series to expand the following function:

   (a) $f(x)=\cos(ax), with -\pi<x\le\pi$ and $a\neq$ integer (15%)

   (b) if $x=\pi$, please show that: $\cot(x)=\sum_{n=-\infty}^{n=\infty}\frac{1}{x+n\pi}$, n=integer (10%)

   There are some useful formulae:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}, with -L<x\le L$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,\ n=0,1,2,3\cdots \quad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx,\ n=1,2,3\cdots$$

$$2\cos\alpha\cos\beta=\cos(\alpha-\beta)+\cos(\alpha+\beta)$$

$$\sin(a\pm n)\pi=(-1)^n\sin a\pi, if\ n=\text{integer}$$