

試題中， $A, B$  代表矩陣。 $A^T$  代表  $A$  的轉置矩陣。 $A^{-1}$  代表  $A$  的反矩陣。

1~4 題為簡答題，每題 5 分，只須回答 True 或 False：

1. If  $A$  is  $n$  by  $n$ , then  $A$  and  $A^{-1}$  have the same determinant.

2. If  $A$  is invertible, then  $\det(A^2) = (\det A)^2$ .

3. If  $A^{-1}$  is positive definite, then  $A$  is also positive definite.

4. If  $A$  and  $B$  are 3 by 3 matrices, then  $(AB)^T = A^T B^T$ .

5~12 題為計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

5.  $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . Find  $A^{-1}$ .

6. Let  $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ . Find a basis for the row space of  $A$ .

7. Are the vectors  $(1,2,1)$ ,  $(0,1,2)$  and  $(3,8,7)$  linearly independent?

8.  $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 2 \end{bmatrix}$  Find the dimension of the column space of  $A$ .

9. Is  $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$  diagonalizable (可對角化)?

10. Is  $\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$  positive definite?

11. Is  $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$  similar to  $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ ?

12. Prove: If  $A$  is a 2 by 2 projection matrix, then the eigenvalues of  $A$  are 0 and 1.