中國文化大學 103 學年度碩士班考試入學招生考試

系所組：化學工程與材料工程學系奈米材料碩士班

日期節次：103 年 3 月 15 日第 1 節 9:00~10:30

科目：工程數學

1. (50%) Solve y(x) for the following differential equations.
   (a) $y''-y=0$ with $y(0)=6$ and $y'(0)=-2$ (15%)
   (b) $y''+4y=2\sin 2x$ with $y(0)=1$ and $y'(0)=0.5$ (20%)
   (c) $xy^2\,dx+(2+x^2y)\,dy=0$ with $y(1)=2$ (15%)
2. (10%) Derive the Laplace transform for the function {sin *kt*} is

$$\mathcal{L}\{\sin kt\}=\frac{k}{s^2+k^2}$$

3. (15%) matrix M=$\begin{pmatrix} 5 & -5 & 0 \\ -1 & 2 & -1 \\ 0 & -5 & 5 \end{pmatrix}$, Please find the three eigenvalues of the matrix M

4. (25%) Using the Fourier series to expand the following function:
   (a) $f(x)=\cos(ax), with -\pi < x \le \pi \; and \; a \neq \text{integer}$ (15%)

   (b) if $x=\pi$, please show that: $\cot(x)=\sum_{n=-\infty}^{n=\infty}\frac{1}{x+n\pi}$, n=integer (10%)

   There are some useful formulae:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}, with -L < x \le L$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx, n=0,1,2,3\cdots \quad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx, n=1,2,3\cdots$$

$$2\cos\alpha\cos\beta=\cos(\alpha-\beta)+\cos(\alpha+\beta)$$

$$\sin(a\pm n)\pi=(-1)^n\sin a\pi, if\ n=\text{integer}$$