

中國文化大學 99 學年度轉學招生考試

系組：經濟學系二、資訊管理學系二年級

日期節次：7 月 28 日 第 3 節 13:30-14:50

科目：微積分 (141-108)

U-5-2

Transfer Test, Calculus

**Show all works to get full credits!**

1. (15%) Evaluate the following integral

$$\begin{matrix} (5\%) \\ (a) \end{matrix} \int x \ln x \, dx \quad \begin{matrix} (5\%) \\ (b) \end{matrix} \int x^2 e^{x^3+1} \, dx \quad \begin{matrix} (5\%) \\ (c) \end{matrix} \int_2^{\infty} \frac{1}{x \ln x} \, dx$$

2. (15%) Determine where the function

$$f(x) = 3x^4 - 2x^3 - 12x^2 + 18x + 15$$

is increasing and decreasing, and where its graph is concave up and concave down.

3. (15%) Find

$$\iint_D 12x \, dA$$

where  $R$  is the region bounded by  $y = x^2$  and  $y = 6 - x$ .

4. (15%) Differentiate these functions

$$\begin{matrix} (7\%) \\ (a) \end{matrix} f(x) = e^{x^2} \ln(x^3 + 4) \quad \begin{matrix} (8\%) \\ (b) \end{matrix} g(x) = \frac{\sqrt{x+3}}{(1-2x)^6}$$

5. (15%) Determine whether each of the following series converges or diverges

$$\begin{matrix} (5\%) \\ (a) \end{matrix} \sum_{k=1}^{\infty} \frac{\ln k}{k} \quad \begin{matrix} (5\%) \\ (b) \end{matrix} \sum_{k=3}^{\infty} \frac{k^2}{5^k} \quad \begin{matrix} (5\%) \\ (c) \end{matrix} \sum_{k=1}^{\infty} \left(\frac{\pi}{2}\right)^k$$

6. (15%) Find all critical points for the function  $f(x, y) = x^3 - y^3 + 6xy$  and classify each as a relative maximum, a relative minimum, or a saddle point.

7. (10%) Find the maximum and minimum values of the function

$$f(x, y) = xy$$

subject to the constraint  $x^2 + y^2 = 8$ .