中國文化大學 99 學年度轉學招生考試

系組：應用數學系二、物理學系二、化學系二、大氣科學系二年級　　日期節次：7 月 27 日第 3 節 13:30-14:50

U-3-1

科目：**微積分**　(119-20)

一、填充題，每格 5 分，共 60 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1. $\lim_{x\to 7}\dfrac{\sqrt{x+2}-3}{x-7}=$ _____.

2. $\lim_{x\to -\infty}\dfrac{2x+1}{\sqrt{x^2+3}}=$ _____.

3. $\dfrac{d}{dx}x^x=$ _____.

4. $\int_0^3 \dfrac{dx}{x-1}=$ _____.

5. $\int_0^{1/\sqrt{3}} \dfrac{dx}{1+3x^2}=$ _____.

6. $\int_0^2 \int_{y/2}^1 e^{x^2}\,dxdy=$ _____.

7. If $F(x)=\int_1^x f(t)dt$, where $f(t)=\int_1^{t^2}\dfrac{\sqrt{1+u^4}}{u}du$, find $F''(2)=$ _____.

8. The area of the region enclosed by $x=y^2$ and $y=x-2$ is _____.

9. If $f(x)=x^5+2x+1$, then $\left(f^{-1}\right)'(4)=$ _____.

10. The slope of the cycloid $x=2(\theta-\sin\theta)$, $y=2(1-\cos\theta)$ at the point where $\theta=\dfrac{\pi}{3}$ is _____.

11. The interval of convergence of the power series $\sum_{n=0}^{\infty}\dfrac{x^n}{(n+1)2^n}$ is _____.

12. The directional derivative of $f(x,y)=x^2y^3-4y$ at $P_0(2,-1)$ in the direction $\mathbf{v}=2\mathbf{i}+5\mathbf{j}$ is _____.

二、計算證明題，每題 10 分，共 40 分。(請寫下計算證明過程，否則不予計分。)

1. Show that the equation $x^3+2x-1=0$ has **exactly one** real solution.

2. Find the Taylor series generated by $f(x)=e^{x^2}$ at $a=0$. Then, approximate the integral $\int_0^1 e^{x^2}dx$ by using the Taylor polynomial of order 2.

3. Find all relative extrema and saddle points of $f(x,y)=3x^3+y^2-9x+4y$.

4. Evaluate the integral $\iint_R e^{\frac{x+y}{x-y}}dA$, where R is the trapezoidal region with vertices (1, 0), (2, 0), (0,-2), and (0, -1).