系所組：化學工程與材料工程學系奈米材料碩士班甲組

日期節次：101 年 3 月 17 日第 3 節 13:00~14:30

科目：工程數學

1. (50%) Solve y(x) for the following differential equations.
   (a) $y''+y'+0.25y=0$ with $y(0)=3$ and $y'(0)=-3.5$ (15%)
   (b) $y''+y=0.001x^2$ with $y(0)=0$ and $y'(0)=1.5$ (20%)
   (c) $\cos(x+y)\,dx+(3y^2+2y+\cos(x+y))\,dy=0$ (15%)
2. (10%) Derive the Laplace transform for the function $\{\cos kt\}$ is

$$\mathcal{L}\{\cos kt\}=\frac{s}{s^2+k^2}$$

3. (20%) matrix $M=\begin{pmatrix} \alpha & 0 & 5\beta \\ 0 & \alpha & 12\beta \\ 5\beta & 12\beta & \alpha \end{pmatrix}$, the three eigenvalues of the matrix M are −7, 6, and 19. Please find the α and β [assume $\alpha>0$ and $\beta>0$]
4. (20%) Find the particular solution of following differential equation:

$$\begin{cases} \frac{dx}{dt}=3x+y+3e^t \\ \frac{dy}{dt}=x+3y \end{cases}$$ which satisfies the initial conduction $\begin{cases} x(0)=5 \\ y(0)=3 \end{cases}$

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