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可移動式機器人混模控制設計
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可移動式機器人混模控制設計

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中　華　民　國 98 年 9 月 10 日

行政院國家科學委員會專題研究計劃成果報告

可移動式機器人混模控制設計
Hybrid-Based Control Designs for Mobile Robots

計劃編號：NSC - 97－2221－E－034－019
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一、中文摘要

本計畫主要目的在克服一般以回步法設計可移式機器人控制器之兩個缺失:1.在所謂之運動設計階段，多數設計不可避免的含有一參考速度之高次方多項式函數，將在高速應用中導致輸入力矩過高因而造成馬達之燒毀；2.在所謂動力設計階段，多數設計皆假設輸入矩陣為完全已知，因而限制了其實用性。針對第一項缺失，我們以一修正之飽和函數取代現有高次多項式函數，此函數其有限輸出的特性有效克制現有設計快速發散之缺點。針對第二項缺失，我們推廣過去平滑切換控制設計成果至可移式機器人系統，有效克服上述第二項問題。除了以理論證明所提設計的有效性，同時以模擬驗證其實用性。

關鍵詞 混模控制器、切換機制、修正式飽和函數。

Abstract

The objective of this project is to conquer two major drawbacks shared by many existing backstepping-based control schemes, first, the kinematic controller contains a high-order polynomial function of the desired velocity which may result in extremely high control torques in fast motion applications; second, the dynamic controllers are restricted for cases with the input matrix being exactly known *a priori*. For conquering the first drawback, a modified saturation function, replacing the high-order polynomial, is included to keep the control from growing unbounded while preserve the asymptotic tracking stability simultaneously. Next, based on our earlier smooth switching control designs, a hybrid control scheme for conquering the second restriction is constructed. Not only the theoretical analysis ensuring the validity of the proposed design is conducted but also simulation results demonstrating the usefulness are also provided

Keywords: Hybrid-based controller, switching mechanism, modified saturation function.

一、簡介

Tracking control of mobile robot systems under nonholonomic constraints, due to its great potential in a wide variety of applications, has received a lot of attentions recently. Numerous schemes, falling into the span of the discontinuous control, the hybrid control, and the backstepping designs, have been proposed to attain the control objectives (see [1] for a review).

A backstepping based controller was first presented in [4] to achieve the semiglobal asymptotic tracking stability for a specific wheeled mobile robot with two degrees of freedom. The smooth time-varying dynamic stabilizer in [5] ensures the global asymptotic stability for multi-input chained systems. To widen its applicability, the dynamic design level, which takes the dynamics into account and aims to develop torque control algorithms, should be initiated.

Regarding this, an adaptive backstepping approach was developed for nonholonomic dynamic systems with inertia parametric uncertainty in [6]. The scheme in [7] ensures the exponential tracking stability on a mild PE condition for a specific type of mobile robots. However, the inclusion of high-degree polynomials of the affine functions in most of these controllers may lead to the possible blowup of the actuators for high-order kinematic systems in high-speed motions.

We intend to conquer such drawbacks in this paper. The control structure here is similar to the one in [5], however, instead of the high-degree polynomial, an exponentially modulated linear stabilizing function is included in our kinematic controller. The modulation function acts to provide a faster convergence to zero than the denominator of the virtual controller and thus prevents the occurrence of singularity, while the linear stabilizing component avoids the blowup of the actuators on the other hand. Next in the dynamic stage, an adaptive control algorithm is developed to achieve the global asymptotic tracking stability of the overall closed-loop system in the presence of the inertia parametric uncertainty.

二、問題簡述

Let $q \in R^n$ denote the generalized coordinate vector of a mobile robot. The corresponding velocities, when subjected to nonholonomic constraints, satisfy

$$J(q)\dot{q} = 0 \tag{1}$$

where $J(q) \in R^{(n-m-1)\times n}$ is the full-rank constraint matrix. Next, the robot dynamics is described by [7]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau + J^T\lambda \tag{2}$$

where $M(q) \in R^{n\times n}$ is the symmetric, positive definite inertia matrix; $\tau \in R^r$ is the available motor torque vector and $B(q)$ is an $n \times r$ full-rank matrix; $\lambda \in R^{n-m-1}$ represents the constraint force; $C(q,\dot{q})\dot{q}$ is the centripetal and Coriolis torque vector while $G(q)$ is the gravitational torque. The following properties known to hold for a general robot are summarized here for the ease of reference.

P1): The left-hand side of (2) is linear in the physical parameters (masses, moment of inertia, etc.) and therefore can be written in a compact form of

$$M(q)\dot{v} + C(q,\dot{q})v + G(q) = H(q,\dot{q},v,\dot{v})\beta \tag{3}$$

where $\beta \in R^p$ denotes the lumped parameter vector while $H \in R^{n\times p}$ is the regression matrix depending on $q, \dot{q}, v$ and $\dot{v}$.

P2): The selection of the matrix $C(q,\dot{q})$ is not unique, and in particular, it can always be selected to render the matrix $\dot{M}-2C$ skew symmetric.

Under the condition that the parameter vector β being unknown, the control objective is to determine a control law for τ such that $(q,\dot{q}) \to (q_d, \dot{q}_d)$ as $t \to \infty$.

The first step is to obtain the dynamics on the reduced constraint manifold fulfilling (1), which is $(m+1)$ dimensional and free from constraint forces. The assumption of $J(q)$ being of full rank implies the existence of a smooth distribution, denoted $J^{\perp}$, which totally annihilates the row vectors of $J(q)$ for all $q \in R^n$. More formally, there exists a set of linearly independent vector field $v(q) \in R^{m+1}$ such that

$$\dot{q} = R(q)v(q) \tag{4}$$

Taking time derivatives of (5) results in

$$\ddot{q} = R\dot{v} + \dot{R}v \tag{5}$$

By substituting (5) and (6) into (2) and then multiplying both sides by $R^T S$, it yields

$$M_1\dot{v} + C_1(q,\dot{q})v + G_1(q) = B_1(q)\tau \tag{6}$$

where $M_1 = R^T M(q) R$,
$C_1(q,\dot{q}) = R^T M(q)\dot{R}(q) + R^T C(q,\dot{q})R(q)$,
$G_1 = R^T G(q)$, and $B_1 = R^T B(q)$.

Equations (4) and (6) constitute a set of $(n+m+1)$ algebraic-differential equations describing the dynamics on the constraint manifold. It is quite common to first convert (4) into certain canonical forms to facilitate the control designs [8]. In the sequel, we assume there exists a diffeomorphic coordinate transformation $y = \phi(q),\ u = \varphi(q)v$ with $\varphi(q) \in R^{(m+1)\times(m+1)}$, under which the kinematic subsystem (4) can be transformed into the m-chain *single-generator chained* canonical form

$$\begin{aligned} \dot{y}_0 &= u_0, \\ \dot{y}_{j,i} &= u_0 y_{j+1,i}, \\ \dot{y}_{n_i,i} &= u_i, \quad 1 \le i \le m, 1 \le j \le n_i - 1 \end{aligned} \tag{7}$$

where n_i is the number of states of the i'th chain with $\sum_{i=1}^{m} n_i + 1 = n$

$$y = [y_0, y_{1,1}, \cdots, y_{n_1,1}, \cdots, y_{1,m}, \cdots, y_{n_m,m}]^T \in R^n$$

is the transformed state vector, and $u = [u_0, \cdots, u_m]^T \in R^{m+1}$ is the corresponding control input vector.

Within such a framework, the dynamic model can be rewritten as

$$\mathrm{M}(y)\dot{u} + C(y,\dot{y})u + G(y) = B(y)\tau \tag{8}$$

where $M(y) = \varphi(q)^{-T} M_1(q)\varphi(q)^{-1}\Big|_{q=\phi^{-1}(y)}$,

$C(y,\dot{y}) = \varphi(q)^{-T} C_1(q,\dot{q})\varphi(q)^{-1}\Big|_{q=\phi^{-1}(y)}$,

$G(y) = \varphi(q)^{-T} G_1(q)\Big|_{q=\phi^{-1}(y)}$,

$B(y) = \varphi(q)^{-T} B_1(q)\Big|_{q=\phi^{-1}(y)}$. It is easy to prove that $M(y)$ remains as a symmetric positive definite matrix and

P3) $M(y)\dot{\eta} + C(y,\dot{y})\eta + G(y) = \Phi(y,\dot{y},\eta,\dot{\eta})\beta$ where Φ is some certain known regression matrix depending on $q, \dot{q}, \eta$ and $\dot{\eta}$.

P4) The matrix $\dot{M}-2C$ is skew symmetric.

On the other hand, the desired trajectory q_d should certainly comply with the constraints in a way of

$$\dot{q}_d = R(q_d)v_d \qquad (9)$$

which, via the similar transformation $y_{d,0} = \phi(q_d)$ and $u_d = \varphi(q_d)v_d$ can also be converted into the chained form

$$\begin{aligned} \dot{\varsigma}_0 &= u_{d,0} \\ \dot{\varsigma}_{j,i} &= u_{d,0}\varsigma_{j+1,i}, \\ \dot{\varsigma}_{n_i,i} &= u_{d,i} \ 1 \le i \le m, 1 \le j \le n_i - 1 \end{aligned} \qquad (10)$$

The trajectory tracking task has been converted into a model following problem, i.e, under the condition of the inertia parameters β being unknown, the goal is to seek an adaptive controller such that $y \to \varsigma$ as $t \to \infty$.

三、 控制設計

In this section, a backstepping based control design will be formulated for attaining the objectives.

By subtracting (7) from (10), the dynamics of the kinematic tracking error vector $e = y - \varsigma$ can be obtained as

$$\begin{aligned} \dot{e}_0 &= u_0 - u_{d,0} \\ \dot{e}_{j,i} &= u_{d,0}e_{j+1,i} + (u_0 - u_{d,0})y_{j+1,i} \\ \dot{e}_{n_i,i} &= u_i - u_{d,i}, 1 \le i \le m, 1 \le j \le n_i - 1 \end{aligned} \qquad (11)$$

The following set of error states are defined accordingly

$$\begin{aligned} z_0 &= e_0 \\ z_{j,i} &= e_{j,i} - \alpha_{j-1,i}, 1 \le i \le m, 1 \le j \le n_i \\ \xi_k &= u_{k-1} - u_{b,k-1}, 1 \le k \le m+1 \end{aligned} \qquad (12)$$

where $\alpha = [\alpha_{0,1}, \cdots, \alpha_{n_i-1,1}, \cdots, \alpha_{n_m-1,m}]^T \in R^{n-1}$

and $u_b = [u_{b,0}, \cdots, u_{b,m}]^T \in R^{m+1}$ are the virtual

controllers at disposal. By a direct differentiation and taking (11) into account, it yields

$$\begin{aligned} \dot{z}_0 &= \xi_1 + u_{b,0} - u_{d,0} \\ \dot{z}_{j,i} &= u_{d,0}(z_{j+1,i} + \alpha_{j,i}) + \\ &\quad (\xi_1 + u_{b,0} - u_{d,0})y_{j+1,i} - \dot{\alpha}_{j-1,i} \\ \dot{z}_{n_i,i} &= \xi_{i+1} + u_{b,i} - u_{d,i} - \dot{\alpha}_{n_i-1,i}, 1 \le j \le n_i - 1 \end{aligned} \qquad (13)$$

Next, define

$$r(u_{d,0}) = u_{d,0}[1 - \exp(u_{d,0}/w_s)^{2\bar{n}}] \qquad (14)$$

where $w_s > 0$ is a design constant and $\bar{n}$ is nonnegative satisfying
$\bar{n} \ge \max n_i - 2, 1 \le i \le m.$

The proposed virtual and actual controllers can now be specified as follows

$$\begin{aligned} \alpha_{0,i} &= 0 \\ \alpha_{1,i}(u_{d,0}, e_{1,i}) &= -k_z r(u_{d,0})z_{1,i} \\ \alpha_{j,i}(\bar{u}_{d,0}^{(j-1)}, \bar{e}_{j,i}) &= -z_{j-1,i} - k_z r(u_{d,0})z_{j,i} + h_{j,i} \\ &\quad + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial e_{k,i}} e_{k+1,i}, \ 2 \le j \le n_i - 1 \\ u_{b,0} &= u_{d,0} + \eta \\ u_{b,i} &= u_{d,i} - u_{d,0}z_{n_i-1,i} - k_z z_{n_i,i} \\ &\quad + u_{d,0}\sum_{k=1}^{n_i-1} \frac{\partial \alpha_{n_i-1,i}}{\partial e_{k,i}} e_{k+1,i} + \sum_{k=0}^{n_i-2} \frac{\partial \alpha_{n_i-1,i}}{\partial u_{d,0}^{(k)}} u_{d,0}^{(k+1)} \\ \tau &= B^{+}[\Phi(y, \dot{y}, u_b, \dot{u}_b)\hat{\beta}(t) - k_1\xi - \Lambda] \end{aligned} \qquad (15)$$

where $h_{j,i}$ and $\Lambda = [\Lambda_1, \cdots, \Lambda_{m+1}]^T$ are defined by

$$\begin{aligned} h_{j,i} &= \frac{1}{u_{d,0}} \sum_{k=0}^{j-2} \frac{\partial \alpha_{j-1,i}}{\partial u_{d,0}^{(k)}} u_{d,0}^{(k+1)} \\ \Lambda_1 &= z_0 + \sum_{i=1}^{m}[\sum_{j=1}^{n_i-1} z_{j,i}y_{j+1,i} - \sum_{j=2}^{n_i} z_{j,i}\sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial e_{k,i}} y_{k+1,i}] \\ \Lambda_k &= z_{n_k,k-1}, \quad k = 2, \cdots, m+1 \end{aligned} \qquad (16)$$

while $\eta \in R$ is a dynamic state described by

$$\dot{\eta} = -k_0 \eta - \Lambda_1 \tag{17}$$

The corresponding update algorithm for $\hat{\beta}$ is given by

$$\dot{\hat{\beta}} = -\gamma_a \Phi^T (y, \dot{y}, u_b, \dot{u}_b)\xi \tag{18}$$

where $\gamma_a > 0$ is the update gain.

The function $r(u_{d,0})$ in (14), which plays a key role in the proposed design, has two desired properties

P5) $r(u_{d,0})u_{d,0} \geq 0, \forall u_{d,0} \in R$

P6) $\lim_{u_{d,0}\to 0} u_{d,0}^{-k}(\partial^j r(u_{d,0}) / \partial u_{d,0}^j) = 0,$
$\forall 1 \leq k \leq \bar{n}, 0 \leq j \leq \bar{n}.$

By substituting (16) into (14) and (9), the resulting closed-loop error dynamics becomes

$$\begin{aligned}
\dot{z}_0 &= \xi_1 + \eta \\
\dot{z}_1 &= u_{d,0} z_{2,i} - k_z u_{b,0} r(u_{d,0}) z_{1,\mathrm{i}} + (\xi_1 + \eta) y_{2,i} \\
\dot{z}_{j,i} &= u_{d,0} z_{j+1,i} - u_{d,0} z_{j-1,i} - k_z u_{b,0} r(u_{d,0}) z_{\mathrm{j},\mathrm{i}} \\
&\quad + (\xi_1 + \eta)(y_{j+1,i} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1,i}}{\partial e_{k,i}} y_{k+1,i}) \\
\dot{z}_{n_i,i} &= \xi_{i+1} - u_{d,0} z_{n_i-1,i} - k_z z_{n_i,i} - (\xi_1 + \eta) \\
&\quad (y_{j+1,i} - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{n_i-1,i}}{\partial e_{k,i}} y_{k+1,i}), \\
M(y)\dot{\xi} &= -k_1 \xi - \Lambda + \Phi(y, \dot{y}, u_b \dot{u}_b)\tilde{\beta} \\
&\quad - \mathrm{C}(\mathrm{y}, \dot{\mathrm{y}})\xi,\ 2 \leq \mathrm{j} \leq \mathrm{n_i} - 1
\end{aligned} \tag{19}$$

The main results are restricted for reference trajectories satisfying the following criterion

A1) y_d is bounded and smooth, and $\lim_{t\to\infty} \inf |u_{d,0}| > 0.$

We can now state that

Theorem 1: Consider the error dynamics in (13), with the control in (15) and the update algorithm in (18). Sustained A1), the following goals can be achieved

- all the signals in the closed-loop system remain bounded;
- the tracking error $e(t) \to 0$ as $t \to \infty$.

四、模擬

To demonstrate the validity of the proposed design, two case studies of a unicycle-like wheeled mobile robot and a fire truck system are conducted in this section.

The constrained dynamics of the unicycle-like wheeled mobile robot in Fig. 1 can be described by [4]

$$J(q)\dot{q} = [\cos q_3 \sin q_3\ 0]\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = 0 \tag{20}$$

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_0 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = \frac{1}{W}\begin{bmatrix} -\sin q_3 & -\sin q_3 \\ \cos q_3 & \cos q_3 \\ L & -L \end{bmatrix}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{21}$$

where q_1, q_2 are the coordinates of the reference point P in the inertial frame, q_3 is the orientation of the reference frame with respect to the inertial frame, m is the mass of the robot, and I_0 is its inertia moment about the vertical axis at point P, W is the radius of the wheels and $2L$ is the length of the axis of the front wheels, and τ_1, τ_2 are the motor torques. The matrix $R(q)$ that spans the $J^{\perp}$ subspace is identified as

$$R(q)=\begin{bmatrix}-\sin q_3 & 0\\ \cos q_3 & 0\\ 0 & 1\end{bmatrix} \quad (22)$$

Clearly, $\dot{q}$ must lie in $J^{\perp}$, which leads to

$$\dot{q}=R(q)v=\begin{bmatrix}-\sin q_3 & 0\\ \cos q_3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix} \quad (23)$$

Following the constructive procedures outlined in [8], the coordinates for a chained form transformation can be found as

$$\begin{bmatrix}y_0\\ y_{1,1}\\ y_{2,1}\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ \cos q_3 & \sin q_3 & 0\\ -\sin q_3 & \cos q_3 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} \quad (24)$$

$$\begin{bmatrix}u_0\\ u_1\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & -(q_1\cos q_3+q_2\sin q_3)\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix} \quad (25)$$

Within such a frame, the kinematic model in (24) can be converted into the following chained form

$$\begin{aligned}\dot{y}_0 &= u_{0,}\\ \dot{y}_{1,1} &= y_{2,1}u_0,\\ \dot{y}_{2,1} &= u_1\end{aligned} \quad (26)$$

For this application, the reduced dynamics on the constraint space in (8) can now be written explicitly as

$$\mathrm{M}(y)\dot{u}+C(y,\dot{y})u=B(y)\tau \quad (27)$$

where $M(y)=[my_{1,1}^2+I_0, my_{1,1}; my_{1,1}, m]$,

$C(y,\dot{y})=[my_{1,1}\dot{y}_{1,1},0; m\dot{y}_{1,1},0]$, and

$B(y)=W^{-1}[y_{1,1}+L, y_{1,1}-L;1,1]$.

Define $\beta=[m,I_0]^T$. By inspecting (27), the corresponding regression matrix can be easily obtained as

$$\Phi=\begin{bmatrix}y_{1,1}^2\dot{u}_{b,0}+y_{1,1}\dot{y}_{1,1}u_{b,0}+y_{1,1}\dot{u}_{b,0} & \dot{u}_{b,0}\\ \dot{u}_{b,0}+\dot{y}_{1,1}u_{b,0}+y_{1,1}\dot{u}_{b,0} & 0\end{bmatrix} \quad (28)$$

The desired trajectory is a circle given by $q_d=[A\cos\omega t\ A\sin\omega t\ \omega t]^T$, where A and ω are positive constants at disposal. By a direct calculation, it is not hard to get that $u_{d,0}=\dot{\varsigma}_0=\omega,$ clearly, A1) fulfills in this application. Moreover, the constraint equation $\dot{q}_d=R(q_d)v_d$ sustains with $v_{d1}=\omega A$ and $v_{d2}=\omega$. By substituting q_d and v_d into (25), it yields

$$\begin{aligned}\varsigma_0 &= \omega t\\ \varsigma_{1,1} &= A\\ \varsigma_{1,2} &= 0\end{aligned} \quad (29)$$

The adopted numerical values in this simulation are $m=2.0, I_0=4.0, W=0.2, L=1.0, A=1.0,$ and $\omega=1.0$. The initial positions and velocities of the wheeled robot are chosen as $q(0)=[0.8,0.6,0.4]^T$. The control gains are given by $k_z=5.0, k_0=2.0, \omega_s=0.02, \gamma_a=0.2$. Due to the lack of PE of the regressor $\Phi(t)$ in the limiting case of $t\to\infty$, the parameter errors do not converge to zero eventually, as shown in Fig. 2 [9]. Nevertheless, the tracking errors still go to zero after about 50 seconds, as depicted in Fig. 3.

五、結論

We have constructed an adaptive controller for the uncertain nonholonomic mobile robot system consisting of the kinematics (1), the dynamics (2), the control algorithm (17), and the parameter update algorithm (21). By adopting an exponentially modulated linear control function instead of the conventional polynomial ones, it preserves the global asymptotic tracking stability without resorting to high control forces at the same time.

六、計畫成果自評

針對本計畫所欲克服之兩個缺失(如摘要)，本成果報告所提之內含修正之飽和函數之控制設計，可成功達成任務，相關成果亦以刊載在國際期刊上 [12]。針對第二項缺失，我們亦有初步具體成果，已整理投稿相關國際期刊，相信在不久將來就會有具體數據呈現，總體而言，本年度的計畫算是有具體成效展現，達成率亦算令人滿意。

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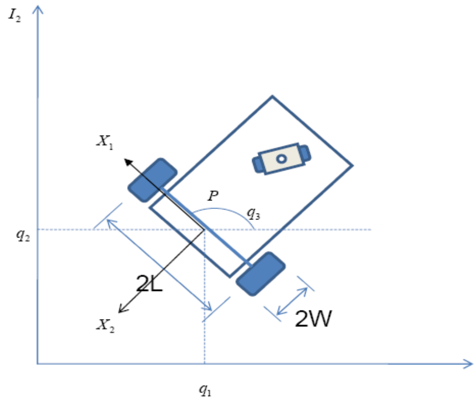

Fig. 1 Schematic diagram of unicycle-like mobile robot

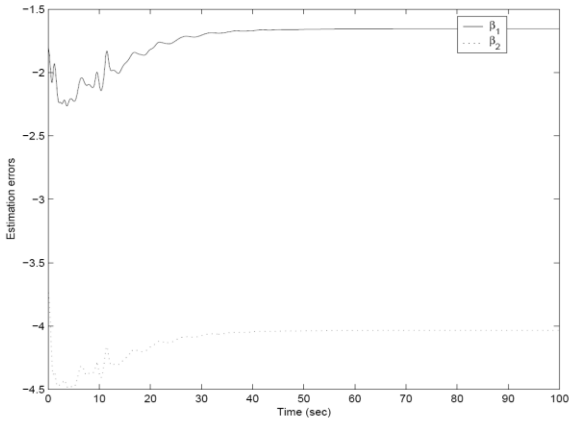

Fig. 2 Estimated parameter errors

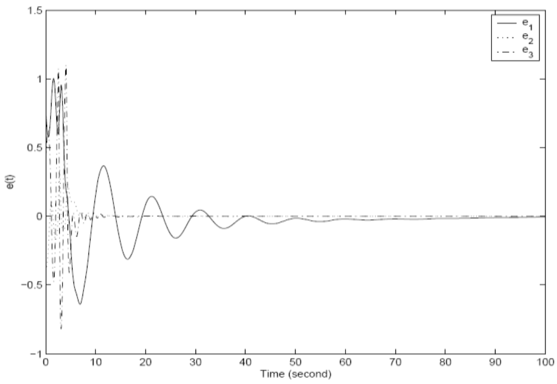

Fig. 3 Tracking error trajectories