

中國文化大學八十六學年度研究所碩士班入學考試

所(組)別： 營工研究所 (丁組)

考試科目： 微 積 分

- Evaluate the following limits. (20%)
 - $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$
 - $\lim_{x \rightarrow \infty} \frac{e^x - x e^{-x}}{e^x + x e^{-x}}$
 - $\lim_{x \rightarrow 0^+} (e^{3x} - 5x)^{1/x}$
 - $\lim_{x \rightarrow 0^+} x^2 \ln x$
- Find the derivative of the following functions.
 - $f(x) = \ln \sqrt{\frac{x}{|x+1|}}$ (5%)
 - $g(x) = \exp(\sqrt[3]{x}) \log_2 5^x$ (10%)
- Evaluate the following definite integrals. (20%)
 - $\int_1^e (\ln x)^2 dx$
 - $\int_0^3 x^2 \sqrt{1+x^2} dx$
- Let the demand equation be given by $p(x) = e^{-0.02x}$ where p is the price per unit. Find x where revenue is maximum. (10%)
- A general population model is $p(t) = \frac{a p_0 (1+t^2)}{a + b p_0 t^2}$ where p_0 is the initial population, i.e., $p(0) = p_0$ and a and b are positive constants. Assume $p_0 < \frac{a}{b}$.
 - Show that $p(t)$ is always increasing for $t > 0$. (5%)
 - Show that the inflection point has t -coordinate $\sqrt{\frac{a}{3b p_0}}$. (10%)
- Suppose a resource, such as oil or gas has total reserves equal to R . Suppose the resource is being extracted at a rate given by $\frac{d}{dt} P(t) = a e^{-kt}$, where a and k are positive constants and t is time. Show that at this rate the resource will be exhausted in time $T = -\frac{\ln(1 - \frac{kR}{a})}{k}$. (10%)
- Suppose a firm has two products X and Y that compete with each other. Let the unit price for X and Y be, respectively, p and q and the demand equations respectively $p(x,y) = 4 - x + 3y$ and $q(x,y) = 8 + x - 5y$ where x and y are the number of items of X and Y, respectively, produced and sold. Find where the revenue attains an absolute maximum for $x \geq 0$ and $y \geq 0$. (10%)

(共一頁)

- 一、(25%)某家烤雞非常有名，排隊買烤雞的人通常很多。現假設若平均要花 20 分鐘排隊才能買到一隻烤雞，且排隊時間呈指數分配，試問：
- Ⓐ(10%) 只花 10 分鐘就可買到的機率？
 - Ⓑ(10%) 排隊半個小時還買不到的機率為何？
 - Ⓒ(5%) 排隊時間介於 15 至 18 分鐘的機率為何？
- 二、(25%) 設 X_1, X_2 是由均值為 μ ，標準差為 σ 的母體中抽取的隨機樣本，若定義三個估計式如下：
- $$\hat{\theta}_1 = \frac{X_1 + X_2}{2}, \hat{\theta}_2 = \frac{2X_1 + X_2}{3}, \hat{\theta}_3 = \frac{2X_1 \cdot X_2}{3}$$
- 試從其中不偏的估計式中，找出具有相對有效性的估計式。
- 三、(10%) 信用卡的使用，近年來在台灣愈來愈普及，根據一項調查指出，台灣地區有 40% 的人至少擁有一張信卡。設若隨機抽取 30 人，則有 18 人至少擁有一張信用卡的機率為何？此題不需算出答案，將最後式子列出即可

四、(40%) 已知

$$n = 32, \bar{X} = 5, \bar{Y} = 40, \sum_i (Y_i - \bar{Y})^2 = 10000, \sum_i (X_i - \bar{X})^2 = 100$$

$$\sum_i (X_i - \bar{X})(Y_i - \bar{Y}) = -800$$

Ⓐ(10%) $\hat{Y} = \hat{\alpha} + \hat{\beta}X$

Ⓑ(10%) 取 $\alpha = 0.05$ ，以 t 檢定法檢定 β 是否為 0？

Ⓒ(10%) 試計算變異數分析表

Ⓓ(10%) 取 $\alpha = 0.05$ 利用 Ⓒ 檢定 β 是否為 0？計算判定係數 (r^2) 之值，並解釋其意義？

$$t_{0.025, 30} = 2.042; F_{0.05(1), 30} = 4.17$$

(第一頁，共一頁)