

- Evaluate the following limits (10%)
 - $\lim_{x \rightarrow 4} \frac{x^2 - 4}{x^2 - 8}$ (5%)
 - $\lim_{x \rightarrow 0} (1 + \sin \frac{3}{x})^x$ (5%)
- Sketch the graph of $y = \frac{\ln x}{x}$ (15%)
- Find the slope of the curve $e^{xy} \ln \frac{x}{y} = x + \frac{1}{y}$ at the point $(e, \frac{1}{e})$ (10%)
- If $H(x) = x \int_4^x e^{-\sqrt{t}} dt$, find $H'(2)$ (10%)
- Evaluate the following integrals
 - $\int \frac{\ln(\ln x)}{x} dx$ (10%)
 - $\int_4^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ (5%)
- If \$2000 is invested at an annual rate of r and compounded monthly, then the amount after t years is given by $A(r, t) = 2000 (1 + \frac{r}{12})^{12t}$. Find $\frac{\partial A}{\partial t}$ (5%)
- Suppose we have a Cobb-Douglas production function $f(x, y) = 100x^{\frac{1}{2}}y^{\frac{3}{4}}$, where x is the number of units of labor, y is the number of units of capital, and f is the number of units of a certain product that is produced. Use the total differential to approximate the change in production as x changes from 16 to 18 and as y changes from 81 to 80. (10%)
- A closed rectangle box of volume 324 in.^3 is to be made with a square base. If the material for the bottom costs twice as much per square foot as the material for the sides and top, find the dimensions of the box that minimize the cost of materials (15%)
- The density function for a town is given by $\delta(x, y) = \frac{120000}{(2+x+y)^2}$ in people per square mile, where x and y are in miles and where the town is the square $D = \{(x, y) | 0 \leq x \leq 2, 0 \leq y \leq 2\}$. Find the total population of this town. (10%)

- 一、(25%)某家烤雞非常有名，排隊買烤雞的人通常很多。現假設若平均要花 20 分鐘排隊才能買到一隻烤雞，且排隊時間呈指數分配，試問：
- ①(10%) 只花 10 分鐘就可買到的機率？
 - ②(10%) 排隊半個小時還買不到的機率為何？
 - ③(5%) 排隊時間介於 15 至 18 分鐘的機率為何？
- 二、(25%) 設 X_1, X_2 是由均值為 μ ，標準差為 σ 的母體中抽取的隨機樣本，若定義三個估計式如下：
- $$\hat{\theta}_1 = \frac{X_1 + X_2}{2}, \hat{\theta}_2 = \frac{2X_1 + X_2}{3}, \hat{\theta}_3 = \frac{2X_1 \cdot X_2}{3}$$
- 試從其中不偏的估計式中，找出具有相對有效性的估計式。
- 三、(10%) 信用卡的使用，近年來在台灣愈來愈普及，根據一項調查指出，台灣地區有 40% 的人至少擁有一張信卡。設若隨機抽取 30 人，則有 18 人至少擁有一張信用卡的機率為何？此題不需算出答案，將最後式子列出即可

四、(40%) 已知

$$n = 32, \bar{X} = 5, \bar{Y} = 40, \sum_{i=1}^n (Y_i - \bar{Y})^2 = 10000, \sum_{i=1}^n (X_i - \bar{X})^2 = 100$$

$$\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) = -800$$

$$\textcircled{1}(10\%) \hat{Y} = \hat{\alpha} + \hat{\beta}X$$

②(10%) 取 $\alpha = 0.05$ ，以 t 檢定法檢定 β 是否為 0？

③(10%) 試計算變異數分析表

④(10%) 取 $\alpha = 0.05$ 利用 F 檢定法檢定 β 是否為 0？計算判定係數 (r^2) 之值，並解釋其意義？

$$t_{0.025, 30} = 2.042; F_{0.05(1), 30} = 4.17$$

(第一頁·共一頁)