1. (a) Show that the following argument form is valid:
p → c, where c is a contradiction
∴ p (5%)

(b) You are visting an island containing two types of people:
Knights who always tell the truth and knaves who always lie. You meet a group of six natives, U,V,W,X,Y,and Z, who speak to you as follows:
U says: None of us is a knight.
V says: At least three of us are knights.
W says: At most three of us are knights.
X says: Exactly five of us are knights.
Y says: Exactly two of us are knights.
Z says: Exactly one of us is a knight.
Which are knights and which are knaves? (5%)

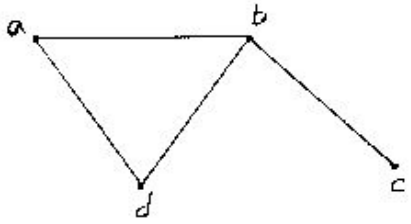
2. Consder the language L specified by the grammar (T,N,S,P) where
T = { a,b,c} is the set of terminals.
N = {S,A,B} is the set of nonterminals.
S is the starting symbol.
P = {S → AB, A →ab, A → aAb, B →c, B → Bc} is the set of production.

(a) Determine whether each of the following strings is a sentence in the language.
aabb
aabbc
aaabbbccc
ababcc (5%)

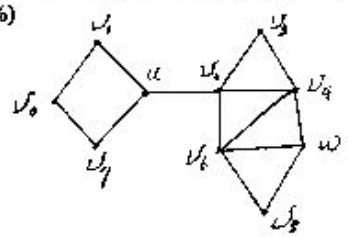
(b) Describe the language L in set-theoretic notation. (5%)

3. A gambler repeatly bets that a die will come up 6 when rolled. Each time the die comes up 6, the gambler wins $1; each time it does not the gambler loses $1. He will quit playing either when he is ruined or when he wins $300. If P_n is the probablity that the gambler is ruined when he begins play with $n, then $P_{k-1} = 1/6 \times P_k + 5/6 \times P_{k-2}$ for all integers k with $2 \leq k \leq 300$. Also $P_0 = 1$ and $P_{300} = 0$. Find an explicit formula for P_n and use it to calculate P_{20}. (10%)

4. (a) Find all possible spanning trees for each of the graphs in the following (5%)

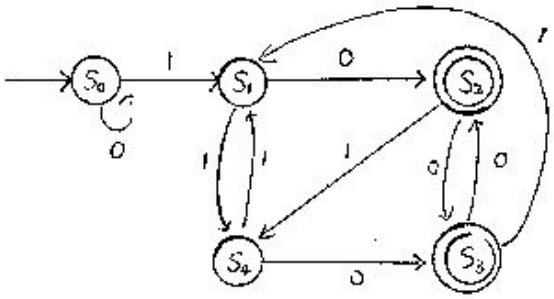


(b) Determine whether there is an Euler path from u to w. If there is, find such a path. (5%)



5. (a) Find the distinct equivalence classes of R.
A = {0,1,2,3,4},
R = {(0,0), (0,4),(1,1),(1,3),(2,2),(3,1),(3,3),(4,0),(4,4)} (5%)

(b) Find the quotient automaton of the automaton shown below. (5%)



6. We used methods of linear regression to predict the average closing bid for each of ten types of used computer equipment based on the average seller's asking price or the average buyer's bid. The data are shown in the following table. (25%)

Machine	Average Seller's Asking Price (seller-p)	Average Buyer's Bid (buyer-b)	Average Closing Bid (close-b)
PC XT	400	200	300
PC AT	700	400	575
XT 089	450	200	325
AT 339	700	350	600
PS/2 30	950	500	725
PS/2 50	1050	700	875
PS/2 70	2000	1600	1725
Compaq	1200	700	875
Gateway	1000	700	900
Dell	1150	800	975

The Minitab incomplete printout shows a multiple regression analysis for the same data. Use the printout to answer the following questions.

Minitab output

```
The regression equation is
close-b = 81.6 + 0.856 seller-p + 0.591 buyer-b

Predictor       Coef      Stdev   t-ratio      p
Constant       81.58      55.12      1.48  0.182
seller-p     0.9564     0.1767      2.02  0.084
buyer-b      0.5915     0.1988      2.98  0.021

s = 41.46      R-sq = 99.1850%     R-sq(adj) = 99.0%

Analysis of Variance

SOURCE       DF          SS          MS          F          p
Regression    2     1465780      732890        XXX
Error         7         XXX         XXX
Total         9

SOURCE       DF      SEQ SS
seller-p      1     1450581    SSR(seller-p)
buyer-b       1       15249    SSR(buyer-b|seller-p)
```

a. What multiple regression model has been fit to the data? What assumptions are necessary in order that our inferences be valid? (5%)

b. Calculate the missing values of F statistics, MSE, and SSE in the ANOVA table. (10%)

c. Do the data provide sufficient evidence to indicate that the model contributes information of y? Test using α=0.05. (5%)

d. Test whether buyer-b can be dropped from the regression model given that seller-p is retained. Use the F test statistics and level of significance 0.05. State the alternative, decision rule, and conclusion. (5%)

7. An automobile manufacturing conducts quality control training programs in two of its supplier plants. The ten instructors give the same exam in both plants. The exam scores are showing as follows. Looking at the Minitab output, answer the following questions. (25%)

Instructor	1	2	3	4	5	6	7	8	9	10
plant (Tainan)	4	3	7	2	1	3	3	1	4	2
plant (Taipei)	5	3	6	2	4	2	6	7	7	5

Minitab output

```
MTB > twosample 85.0 'tainan' 'taipei';
SUBC> alternative 0;
SUBC> pooled.

TWOSAMPLE T FOR Tainan VS Taipei
        N      MEAN     STDEV   SE MEAN
Tainan  10     3.00      1.76      0.56
Taipei  10     4.70      1.89      0.60

 TTEST MU Tainan = MU Taipei (VS NE): T= XXXXX
 POOLED STDEV =        1.83

MTB > let c3='tainan' - 'taipei'
MTB > name c3 'diff'
MTB > ttest 0.0 'diff';
SUBC> alternative 0.

TEST OF MU = 0.000 VS MU N.E. 0.000

             N      MEAN     STDEV   SE MEAN        T    P VALUE
diff        10    -1.700     2.263     0.716     XXXX      XXXX
```

a. Conduct a two-sample t-test on the two plants by using level of significance 0.05. State the hypotheses, decision rule, result and conclusion. (5%)

b. Test, using the paired t-test, whether the two plant's exam score are different or not. Use level of significance 0.05 and state the hypotheses, decision rule, result and conclusion. (5%)

c. Which is the appropriate method to conduct this test? Explain? (5%)

d. Try to explain what is statistical significant and what is practical significant? (10%)

Critical Value of t

d.f.	$t_{.050}$	$t_{.025}$	$t_{.010}$
7	1.895	2.365	2.998
8	1.860	2.306	2.896
9	1.833	2.262	2.821
10	1.812	2.228	2.764
18	1.734	2.101	2.552
19	1.729	2.093	2.539
20	1.725	2.086	2.528

Critical Value of F

$F_{3,9}(0.05) = 4.26$

$F_{5,9}(0.05) = 4.74$

$F_{8,9}(0.05) = 5.12$

$F_{1,5}(0.05) = 5.59$