中國文化大學八十五學年度研究所碩士班入學考試

國際企業管理研究所 | 考試科目：微 積 分

1. Determine the limits

(a) $\lim_{x\to 3} \frac{|x-3|}{2x-6}$ (5%)

(b) $\lim_{\Delta x\to 0} \frac{3-(x+\Delta x)^2-(3-x^2)}{\Delta x}$ (5%)

2. Evaluate

(a) $\int_{-\pi/2}^{\pi/2} (\sin^3 x\cos x+\sin x\cos x)dx$ (5%)

(b) $\int_0^\infty \frac{dx}{\sqrt{x}(x+1)}$ (5%)

3. Evaluate (10%)

$$\int_R\int (1-\frac{1}{2}x^2-\frac{1}{2}y^2)dA$$

where R is the region given by $0\le x\le 1,\quad 0\le y\le 1$

4. Find the volume of the solid of revolution formed by revolving the region bounded by $y=\frac{2}{e^x}$, $y=0$ and $x\ge 0$ about the x-axis (if possible). (10%)

5. Verify the convergence of the following geometric series.

(a) $2-\frac{1}{2}+\frac{1}{8}-\frac{1}{32}+\frac{1}{128}-\cdots$ (5%)

(b) $\sum_{n=0}^{\infty}\left[(0.2)^n-(0.5)^n\right]$ (5%)

Each Question counts 10%.

Answer all questions.

6) Suppose country A's per-capita GNP is 11 times that of country B. However, country B's rate of per-capita income growth is 12% per year, while country A's growth rate is 6% per year. Assuming that these growth rates are stable and continuous, how many years would be required to equalize per-capita incomes in the two countries?

7) The problem with automatic cash machines is that the user pays a service charge for each withdrawl. But if one tries to avoid the problem by making large withdrawls at infrequent intervals, one must carry large amounts of cash and run the risk of loss or theft. Suppose a person has a steady cash expenditure requirement (E) of NT$ 1600 per day, and each withdrawl requires a service charge (C) of NT$ 20. Furthermore, on any specific day, this person has a probability (P) of .004 of losing all his cash through loss or theft. If the person's average cash holding is exactly half of his average cash machine withdrawl amount, what is the optimal amount to withdraw on each visit to the cash machine, and how frequently should he withdraw?

8) Suppose your firm faces a demand curve described by P(Q). It can supply its product from two factories, which have different cost functions: C1(q1) and C2(q2). If Q = q1 + q2, what should the profit-maximizing total output of the firm be, and how much should be supplied from each factory?

$$P(Q) := \frac{1000}{\sqrt{Q}} \qquad C1(q1) := (.02)\cdot q1^2 + (6)\cdot q1 \qquad C2(q2) := (20)\cdot q2$$

9) Let output be a function of capital and labor inputs Q(K,L). What is the most efficient input mix with which to produce a total output of 900 units, if the cost of a unit of capital is 4 times the cost of a unit of labor?

$$Q(K,L) := K^{\left(\frac{1}{3}\right)}\cdot L^{\left(\frac{2}{3}\right)}$$

10) A consumer of two products (X,Y) obtains utility $U(X,Y) := Y\cdot\sqrt{X}$ The consumer's budget remains constant at 120. If the price of good X remains constant at 10, find the gain in the consumer's total utility from a fall in the price of Y from 10 to 8.

(共一頁)