

科目：工程數學[2931]

1. (40%) Solve $y(x)$ form the following ordinary differential equation:

(a) $y''+6y'+9y = 0$, with $y(0) = a_1$ and $y'(0) = -3a_1 + a_2$ (10%)

(b) $2xyy' + y^2 = 4$, with $y(1) = 0$ (10%)

(c) $x^2y'' - 12y = 24x$ (10%)

(d) $y''+4y'+4y = 6e^{-2x}$ (10%)

2. (15%) Use the Laplace transform to solve the differential equations ONLY!!!

$$\begin{cases} \frac{dX(t)}{dt} = -aY(t) \\ \frac{dY(t)}{dt} = 4aX(t) \end{cases} \text{ and } t=0, X = X(0), Y = Y(0), a \in R \text{ 用其他方法不以計分!!!}$$

3. (20%) Define $M = \begin{bmatrix} -1 & 0 & 12 & 0 \\ 0 & -1 & 0 & 12 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & -4 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} \frac{9}{8} \\ -\frac{9}{8} \\ \frac{3}{8} \\ -\frac{3}{8} \end{bmatrix}$.

(a) Find the four eigenvalues ($\lambda_1, \lambda_2, \lambda_3$ and λ_4) of matrix M . (4%)

(b) Assume $\lambda_1 > \lambda_2 = \lambda_3 > \lambda_4$. Find the two eigenvectors of the matrix M , which eigenvalues are λ_1 and λ_4 . (8%)

(c) M^{-1} is the inverse matrix of M . Calculate the $M^{-1}b = ?$ (8%)

4. (25%) Using the Fourier series to expand the following function:

(a) $f(x) = \cos(ax)$, with $-\pi < x \leq \pi$ and $a \neq \text{integer}$ (15%)

(b) if $x = \pi$, please show that: $\cot(x) = \sum_{n=-\infty}^{\infty} \frac{1}{x + n\pi}$, $n = \text{integer}$ (10%)

There are some useful formulae.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \text{ with } -L < x \leq L$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, \quad n = 0, 1, 2, 3, \dots \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, \quad n = 1, 2, 3, \dots$$

$$2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$\sin(a \pm n)\pi = (-1)^n \sin a\pi, \text{ if } n = \text{integer}$$