Fuzzy Inventory without Backorder based on Intuitionistic Fuzzy Sets and Decomposition Theory

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Abstract

In this article, we consider inventory without backorder. In a plan period, to determine the total demand to be a fixed value is difficult in general. Therefore we fuzzify it to be a fuzzy set. Similarly we fuzzify the storing cost. Since the ordinary fuzzy set is a special case of intuitionistic fuzzy set. Hence we use intuitionistic fuzzy set to discuss this problem. In order to fit the real situation, we fuzzify the total demand and storing cost to intuitionistic fuzzy sets and transform into equivalent interval-valued fuzzy sets. Then we fuzzify the total demand and storing cost in the crisp total cost, respectively, to fuzzy total cost. Through signed distance to defuzzify, we get the estimate of the total cost in the fuzzy sense. We can find the optimal order quantity in the fuzzy sense.

Keyword： Fuzzy total cost, Interval-valued fuzzy set, Intuitionistic fuzzy set, Signed distance.

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§1. Introduction

In classical inventory models, the parameters in the total cost are known and fixed. It does not cause any ambiguity. The variables in the total cost are positive and real. But in a real situation, fuzzy phenomena may occur. If this happens, we need to consider fuzzy inventory models. We could consider the following fuzzy inventory models according to different fuzzy situations.

Vujošević etc. [16] fuzzified the order cost in the total cost to trapezoidal fuzzy number in their inventory model without backorder. They got fuzzy total cost and used centroid to defuzzify to get the estimate of the total cost in the fuzzy sense. Cheng and Wang [5] fuzzified order cost, inventory cost and backorder cost in the total cost to trapezoidal fuzzy numbers in their inventory with backorder model. They used functional principle to find the estimate of the total cost in the fuzzy sense. Roy and Maiti [15] formulated the classical economic order quantity problem to a nonlinear programming problem. They fuzzified the objective function and storage area. It is solved by both fuzzy nonlinear and geometric programming techniques for linear membership functions. Ishii and Konno [8] fuzzified shortage cost to L fuzzy number in their classical newsboy problem. They found optimal ordering quantity through the concepts of fuzzy ordering.

Yao etc.〔4, 10~12, 17~20〕, discussed the fuzzy problems concerning inventory with or without backorder and production inventory. In〔10, 17〕, they fuzzified order quantity in the total cost of inventory without backorder to triangular fuzzy number or trapezoidal fuzzy number. Then they used extension principle and centroid to get estimate of the total cost in the fuzzy sense. In〔18〕, they fuzzified order quantity and total demand quantity in the total cost of inventory without backorder to triangular fuzzy number respectively. Then they had fuzzy total cost and they used extension principle and centroid to defuzzify. In〔19〕, they fuzzified the order quantity in the total cost of inventory with backorder to triangular fuzzy number and kept the backorder quantity as a positive real number. Then they got fuzzy total cost and they also used extension principle and centroid to defuzzify. In〔4〕, they fuzzified backorder quantity in the total cost of inventory with backorder to triangular fuzzy number and kept order quantity as positive real variable. In a similar manner, they obtained fuzzy total cost and used extension principle and centroid to defuzzify. In〔20〕, they fuzzified the total demand in the total cost of inventory with backorder to interval-valued fuzzy set. Then they had fuzzy total cost. After defuzzification, they got estimate of total cost in the fuzzy sense. In〔11〕, they fuzzified the production quantity and demand quantity per day in the total cost of production inventory to triangular fuzzy set. Then they got fuzzy total cost and they used extension principle and centroid to solve this problem. In〔10〕, they fuzzified the production quantity per cycle in the total cost of production inventory to triangular fuzzy number. Then they obtained fuzzy total cost and also used extension principle and centroid to defuzzify. In articles〔4, 10~12, 17~19〕, they all used extension principle to find the membership function of the fuzzy total cost. It is very complicated to do so. In this paper, we use intuitionistic fuzzy set to transform to interval-valued fuzzy set (*IVFS*) and fuzzify the storing cost and total demand in the crisp total cost respectively. Then we have fuzzy total cost. Through defuzzification by signed distance of *IVFS*, we obtain estimate of the total cost in the fuzzy sense. It is more convenient to use signed distance to defuzzify than to use extension principle.〔1, 2, 3, 6〕are the articles about intuitionistic fuzzy sets. We define intuitionistic fuzzy set, *IVFS* and the signed distance of *IVFS* in section 2. In section 3, we fuzzify storing cost and total demand in the total cost of inventory without backorder by transforming intuitionistic fuzzy set to interval-valued fuzzy set. In this way, we get fuzzy total cost. Using signed distance and decomposition theorem, we have the estimate of the total cost in the fuzzy sense. It follows that we can obtain the optimal solution. We give an example in section 4. Section 5 is the discussion.

§ 5 Discussion

(5.1) The relation of optimal solution between crisp case and fuzzy case in proposition 2,3.

By remark 4, when $\Delta_1 = \Delta_2, \Delta_3 = \Delta_4, \Delta_5 = \Delta_6 = 0$ or $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4$ and $w_1 = w_2, w_3 = w_4,\ w_5 = w_6 = 0$ or $w_1 = w_2 = w_3 = w_4$, we have $P(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6) = 0$ and $P(w_1, w_2, w_3, w_4, w_5, w_6) = 0$. Then $FTC(q) = F(q)$. From proposition 3, $q_\circ = \sqrt{\frac{2ar}{Tc}} = q_*$. Therefore $FTC(q_\circ) = F(q_*)$.

(5.2) The relation between minimum total cost $FIC(q_\circ)$ in the fuzzy case and minimum total cost $F(q_*)$ in the crisp case.

(a)In proposition 2, if $P(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6) > 0$ and $P(w_1, w_2, w_3, w_4, w_5, w_6) > 0$ then $FTC(q) > F(q),\ \ \forall q > 0$. It follows that $FTC(q) > F(q) \geq F(q_*),\ \ q > 0$. Hence $FTC(q_\circ) > F(q_*)$. We conclude that the minimum total cost $FTC(q_\circ)$ in the fuzzy case is larger than the minimum total cost $F(q_*)$ in the crisp case (see the example, case 5~7 in section 4, Table 1).

(b) In proposition 2, if $P(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6) < 0$ and $P(w_1, w_2, w_3, w_4, w_5, w_6) < 0$ then $FTC(q) < F(q),\ \ \forall q > 0$. Hence $FTC(q_\circ) \leq FTC(q_*) < F(q_*)$ (see the example, case 8~10 in section 4, Table 1).

(5.3) The errors of the fuzzy total cost $FTC(q)$ and the crisp total cost $F(q)$.

By proposition 2,

$$FTC(q) - F(q) = \frac{1}{16} Tq[2(\Delta_2 - \Delta_1) + (\Delta_4 - \Delta_3) + \Delta_5(\Delta_3 - \Delta_1) + \Delta_6(\Delta_2 - \Delta_4)]$$

$$+ \frac{a}{8q}[2(w_2 - w_1) + (w_4 - w_3) + w_5(w_3 - w_1) + w_6(w_2 - w_4)].$$ For any fixed $\Delta_3 - \Delta_1 (< 0),\ \ w_3 - w_1 (< 0)$, if $\Delta_2 - \Delta_1, \Delta_4 - \Delta_3, \Delta_2 - \Delta_4$ and $w_2 - w_1,\ \ w_4 - w_3,\ \ w_2 - w_4$ are positive and $P(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6) > 0$, $P(w_1, w_2, w_3, w_4, w_5, w_6) > 0$, then $FTC(q) > F(q)$ (see the example, cases 5~7 in section 4, Table 1).

For any fixed $\Delta_2 - \Delta_4 (> 0),\ \ w_2 - w_4 (> 0)$, if $\Delta_2 - \Delta_1, \Delta_4 - \Delta_3, \Delta_3 - \Delta_1$ and $w_2 - w_1,\ \ w_4 - w_3,\ \ w_3 - w_1$ are negative and $P(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6) < 0$,

$P(w_1, w_2, w_3, w_4, w_5, w_6) < 0$, then $FTC(q) < F(q)$ (see the example, cases 8~10 in section 4, Table 1).

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