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模糊訂貨量與總需求量之存庫問題

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中文摘要

在傳統存庫問題, 允許缺貨的情況, 設 s爲最大庫存量, a爲每單位每天之庫存費用, c爲每次訂貨成本, q爲每次訂貨量, T爲總計劃之總時間長度 (以天爲單位), r爲計劃期間$[0,T]$之總需求量; 則總成本函數爲 $F(q,s)=\frac{aTs^2}{2q}+\frac{bt(q-s)^2}{2q}+\frac{cr}{q}$, $q>0$, $s>0$. 針對此模式之模糊化, 已有一些文章討論, 概述如下: 在文章[1]中, 是利用模糊集合觀念, 將a,b,c,r,T皆視爲模糊數, 使用數值運算藉著函數原理(function principle) 解此問題; 特別要注意的是這裡的a,b,c,r,T 只是常數(parameters) 而非變數(variables)。而在文章[6]中, 就把傳統模式中的變數q視爲模糊變數但s乃爲一般的變數. 反之, 在文章[2]中, 把傳統模式中的變數s視爲模糊變數, 而q乃爲一般的變數. 感覺上,[6]與[2]只有一個模糊變數, 似乎較[1]簡單, 實則不然; 因爲模糊變數比模糊常數在計算上, 困難很多. 在文章[7]中, 考慮不允許缺貨情況下, 把訂貨量 (變數q) 與總需求量 (常數r) 皆模糊化。本研究計劃是考慮在允許缺貨情況下, 把訂貨量與總需求量皆模糊化再求最適的訂貨量.

由於傳統存庫問題中, a,b,c,s與T皆設爲給定常數, 從訂貨至到貨的期間亦視爲固定已知數, 實際上, 受到交通與其他不可抗拒因素的影響, a,b,c,s與T皆有少許變動, 以致直接或間接的影響到每期需求量q與總需求量r. 同時, 從關係式 "$r=q\times$期數" 來看, 在[6]中把q模糊化, 但r爲定數也不甚合理. 故本計劃將r視爲模糊常數, q視爲模糊變數, 在允許缺貨下求最適訂貨量。在利用擴張原理求總成本函數之隸屬函數時, 由於式中有兩個模糊數, 不但計算過程很繁雜, 連隸屬函數之表示亦頗爲繁瑣; 詳細過程於皆省略, 僅列出一個例子的電腦輸出. 從輸出結果可以看出, 模糊化的情況是傳統問題的推廣.

關鍵詞: 模糊存庫問題, 隸屬函數, 擴張原理, 最適訂貨量.

英文摘要

In the classical inventory with backorder model, the time period between the ordering and arriving of stock per cycle is in general not the same in practical situations. It will fluctuate a little. This will influence the ordering quantity q per cycle. Therefore we fuzzify q to a triangular fuzzy number $\tilde{Q} = (q_1, q_0, q_2)$, $0 < q_1 < q_0 < q_2$. It is a little bit hard to find r_0 for the total demand r in the plan period. In real situation, the total demand will be round r_0. We have fuzzy language " about r_0". Hence we fuzzify r to a triangular fuzzy number $\tilde{R} = (r_1, r_0, r_2)$, $0 < r_1 < r_0 < r_2$. Therefore, we consider the inventory problem with backorder such that both order and total demand quantities are triangular fuzzy numbers respectively. Let s denote the maximum inventory quantity. Under conditions $s \leq q_1 < q_0 < q_2 < r_1 < r_0 < r_2$, we find the membership function $\mu_{G(\tilde{Q},\tilde{R})}(z)$ of the total fuzzy cost function $G(\tilde{Q}, \tilde{R})$ and their centroid, then obtain the order quantity q^* in the fuzzy sense and the estimate of the total demand quantity r^*.

From a numerical example, we let $a = 20$, $b = 5$, $c = 30$, $r_0 = 300$ and $T = 10$, then $q_* = 21.21$,$s_* = 4.24$ are the crisp optimal order quantity and optimal maximum inventory quantity. $F(q_*, s_*) = 848.5281$ is the crisp minimum total cost. For the fuzzified case, we know that when s is near to s_*; q_1, q_0, and q_2 are near to q_*; also, r_1, and r_2 are near to r_0, then the results of fuzzy case and crisp case are closed.

Keyords: Fuzzy inventory with backorder; Membership functions; Extension principle; Fuzzy economic order quantity.

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報告內容

When we discuss the classical inventory with backorder model, we get the total cost function $F(q,s)=\frac{aTs^2}{2q}+\frac{bt(q-s)^2}{2q}+\frac{cr}{q}$, $q>0$, $s>0$; where a is the cost of storing one unit for one day, c is the order cost per cycle, q is the order quantity per cycle, T is the planning time for the whole period and r is the total demand over the planning time period $[0,T]$. Some fuzzy inventory problems have been discussed in [1]~[7]. Chen and Wang [1] used fuzzy set concepts in the model. They replaced the costs a, b, c and R which are parameters but not variables in $F(q,s)$ by fuzzy numbers. Also, they solved the fuzzy order quantity problem with numerical operation based on the function principle. Yao and Lee [7] solved fuzzy order quantity problem by fuzzifying q to a fuzzy number with s an ordinary positive real number. Chang, Yao and Lee [2] used extension principle to solve fuzzy order quantity problem by fuzzifying s to a fuzzy number with q an ordinary positive real number. Yao etc.[7] solved fuzzy order quantity and fuzzy total demand quantity in inventory without backorder.

In classical inventory with backorder model, a, b, c, T and r are given constants. Because the total cost on the planning time period $[0,T]$ is given by

$$F(q,s)=[a\times t_1\times\frac{s}{2}+b\times t_2\times\frac{q-s}{2}+c]\frac{r}{q}=\frac{as^2T}{2q}+\frac{b(q-s)^2T}{2q}+\frac{cr}{q}$$

Then, we obtain

$$q_*=\sqrt{\frac{2(a+b)cr}{abT}} \text{ : the crisp economic order quantity}$$

$$s_*=\sqrt{\frac{2bcr}{a(a+b)T}} \text{ : the crisp economic backorder quantity}$$

$$F(q_*,s_*)=\sqrt{\frac{2abcrT}{a+b}} \text{ : the minimum total cost}$$

For the total cost function $F(q,s)$, consider q,r are variables and denote

$$G_s(q,r)=\frac{as^2T}{2q}+\frac{b(q-s)^2T}{2q}+\frac{cr}{q},\ q>0,\ r>0$$

be the total cost function for given s.

Since the order quantity q may slightly change for some uncontrollable situations (as mention in the introduction). Therefore, corresponding to the crisp order quantity $q_0(>0)$ which is an unknown number, we consider the fuzzy order quantity $\tilde{Q} = (q_0 - \Delta_1, q_0, q_0 + \Delta_2)$ which may change around q_0, where $0 < \Delta_1 < q_0$, $0 < \Delta_2$. For given s, suppose the membership function of $\tilde{Q}$ is given by

$$\mu_{\tilde{Q}}(q) = \begin{cases} \frac{q-q_1}{q_0-q_1} & q_1 \le q \le q_0 \\ \frac{q_2-q}{q_2-q_0} & q_0 \le q \le q_2 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

where $q_1 = q_0 - \Delta_1$, $q_2 = q_0 + \Delta_2$, $0 < \Delta_1 < q_0 - s$, $0 < \Delta_2$, and Δ_1 ,Δ_2 are determined by the decision maker based on the uncertainty of the problem. Then

$$M_Q(q_1, q_0, q_2) = \frac{1}{3}(q_1 + q_0 + q_2) = q_0 + \frac{\Delta_2 - \Delta_1}{3} \tag{2}$$

is the centroid of $\mu_{\tilde{Q}}(q)$. Also, the total demand r is inherently extremely difficult to estimate without any error. Therefore, we consider the total demand quantity r is a fuzzy numbers $\tilde{R}$ near r_0 with membership function $\tilde{R}$ given by

$$\mu_{\tilde{R}}(r) = \begin{cases} \frac{r-r_1}{r_0-r_1} & r_1 \le r \le r_0 \\ \frac{r_2-r}{r_2-r_0} & r_0 \le r \le r_2 \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

where $s \le q_1 < q_0 < q_2 < r_1 < r_0 < r_2$ (r_0 is a known constant). Similarly,

$$M_R(r_1, r_0, r_2) = \frac{1}{3}(r_1 + r_0 + r_2) \tag{4}$$

is the centroid of $\mu_{\tilde{R}}(r)$.

Let $G_s(q, r) = y$ (≥ 0), then

$$bTq^2 - 2(y + bTs)q + (a+b)Ts^2 + 2cr = 0$$

and hence

$$r = \frac{-bTq^2 + 2(y + bTs)q - (a+b)Ts^2}{2c}.$$

Also,

$$r \geq 0 \Leftrightarrow y \geq \frac{bTq^2 - 2bTsq + (a+b)Ts^2}{2q} \equiv g(q) \text{ (say)} \forall q \in [q_1, q_2]$$

Thus, $g(q)$ is minimized if $q = \sqrt{\frac{a+b}{b}}s$ and $g(q)$ is maximized if $q = q_1$ or $q = q_2$. Furthermore,

$$g(q_1) \leq g(q_2) \Leftrightarrow q_1 q_2 \geq \frac{(a+b)s^2}{b}$$

Let

$$t_*(s, q_1, q_2) = \max[g(q_1), g(q_2)] = \begin{cases} g(q_2) & q_1 q_2 \geq \frac{(a+b)s^2}{b} \\ g(q_1) & q_1 q_2 < \frac{(a+b)s^2}{b} \end{cases}$$

Therefore, for every $q \in [q_1, q_2]$, the range of $y \geq g(q)$ is equivalent to $y \geq t_*(s, q_1, q_2)$. By the extension principle, for $y \geq t_*(s, q_1, q_2)$. The membership function of the fuzzy total cost function $G_s(\tilde{Q}, \tilde{R})$ is given by

$$\begin{aligned} \mu_{G_s(\tilde{Q},\tilde{R})}(y) &= \underset{(q,r)\in G_s^{-1}(y)}{sup} \left[\mu_{\tilde{Q}}(q) \bigwedge \mu_{\tilde{R}}(r)\right] \\ &= \bigvee_{q_1<q<q_2} \left[\mu_{\tilde{Q}}(q) \bigwedge \mu_{\tilde{R}}\left(\frac{-bTq^2 + 2(y+bTs)q - (a+b)Ts^2}{2c}\right)\right] \end{aligned} \tag{5}$$

By (3) we have

$$\mu_{\tilde{R}}\left(\frac{-bTq^2 + 2(y+bTs)q - (a+b)Ts^2}{2c}\right) = \begin{cases} f_2(q) & u_2 \leq q \leq u_0 \\ f_1(q) & u_0 \leq q \leq u_1 \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

Where

$$f_1(q) = \frac{-bTq^2 + 2(y+bTs)q - (a+b)Ts^2 - 2cr_1}{2c(r_0 - r_1)},$$

$$f_2(q) = \frac{bTq^2 - 2(y+bTs)q + (a+b)Ts^2 + 2cr_2}{2c(r_2 - r_0)}$$

and $u_j = \frac{1}{bT}\left[\sqrt{y^2 + 2bTsy - abT^2s^2 - 2bcTr_j} + y + bTs\right]$

$$\text{if } y \geq -bTs + \sqrt{(a+b)bT^2s^2 + 2bcTr_j}, \quad j = 1, 0, 2$$

Since

$$-bTs + \sqrt{(a+b)bT^2s^2 + 2bcTr_j} < -bTs + \sqrt{(a+b)bT^2s^2 + 2bcTr_2} \equiv E_*(s) \text{ (say)}$$

so $y \geq -bTs + \sqrt{(a+b)bT^2s^2 + 2bcTr_j}$ for all $j = 1, 0, 2$ if $y \geq E_*(s)$, and hence y must satisfy $y \geq E_*(s)$ and $y \geq t_*(s, q_1, q_2)$. In order to find the range of y and then to obtain the solution of equation (6), we denote

$$e_1(s, q_1, q_2) = \max[E_*(s), t_*(s, q_1, q_2)].$$

For $y \geq e_1(s, q_1, q_2) = \max[E_*(s), t_*(s, q_1, q_2)]$, we consider $\mu_{G_s(\tilde{Q},\tilde{R})}(y)$ in equation (5) under conditions 1° and 2° as following cases:

case 1: $u_0 \leq q_0$ and $u_1 \geq q_1$,

case 2: $u_0 \geq q_0$ and $u_2 \leq q_2$.

We use the extension principle to find the membership function $\mu_{G_s(\tilde{Q},\tilde{R})}(y)$ of the fuzzy cost $G_s(\tilde{Q}, \tilde{R})$ and its centroid $M(s, q_1, q_0, q_2, r_1, r_2)$ which will be an estimate of the total cost. Then we will obtain the order quantity q^* in the fuzzy sense and the estimate of the total demand quantity r^*.

Let $a = 20$, $b = 5$, $c = 30$, $r_0 = 300$ and $T = 10$, then $q_* = 21.21, s_* = 4.24$ are the crisp optimal order quantity and optimal maximum inventory quantity. $F(q_*, s_*) = 848.5281$ is the crisp minimum total cost.

In the fuzzy sense, we use a FORTRAN program to find $M^* = M_{jik}(s, q_1, q_0, q_2, r_1, r_2)$, which is an estimate of the total cost under the fuzzy order quantity (q_1, q_0, q_2), maximum inventory quantity s and the fuzzy total demand quantity (r_1, r_0, r_2). Also, let $q^* = \frac{q_1+q_0+q_2}{3}$ and $r^* = \frac{r_1+r_0+r_2}{3}$ be the centroid of (q_1, q_0, q_2) and (r_1, r_0, r_2) respectively. Then the relative error in fuzzy sense for order quantity, total demand quantity and total cost are given by

$$Rel\ Q = \frac{q^* - q_*}{q_*}, \quad Rel\ R = \frac{r^* - r_0}{r_0}, \quad Rel\ C = \frac{M^* - F(q_*, s_*)}{F(q_*, s_*)}$$

respectively. For some sets of (q_1, q_0, q_2) and (r_1, r_0, r_2), we have the numerical results in Table 1. From Table 1, we know that when s is near to s_*, q_1, q_0 , and q_2 are near to q_*. when r_1, and r_2 are near to r_0, then the results of fuzzy case and crisp case are close, i.e., *Rel* C is small whenever *Rel* Q and *Rel* R are small.

Table 1

Numerical Results for $a = 20$, $b = 5$, $c = 30$, $r_0 = 300$, $T = 10$.

s	q_1	q_0	q_2	r_1	r_2	q^*	r^*	M^*	*Rel Q*	*Rel R*	*Rel C*
3.51	20.11	22.71	25.11	296.2	302.2	22.65	299.47	860.98	.0676	-.0018	.0147
3.51	19.41	22.01	24.41	297.6	300.1	21.95	299.23	856.37	.0346	-.0026	.0092
4.03	20.11	22.71	25.11	296.2	302.2	22.65	299.47	857.11	.0676	-.0018	.0101
4.03	19.51	22.11	24.51	297.4	300.4	22.05	299.27	853.30	.0393	-.0024	.0056
4.07	19.81	22.41	24.81	296.8	301.3	22.35	299.37	854.96	.0534	-.0021	.0076
4.07	19.61	22.21	24.61	297.2	300.7	22.15	299.30	853.65	.0440	-.0023	.0060
4.11	19.71	22.31	24.71	297.0	301.0	22.25	299.33	854.20	.0487	-.0022	.0067
4.11	19.41	22.01	24.41	297.6	300.1	21.95	299.23	852.18	.0346	-.0026	.0043
4.19	19.71	22.31	24.71	297.0	301.0	22.25	299.33	853.89	.0487	-.0022	.0063
4.19	19.51	22.11	24.51	297.4	300.4	22.05	299.27	852.58	.0393	-.0024	.0048
4.22	19.61	22.21	24.61	297.2	300.7	22.15	299.30	853.19	.0440	-.0023	.0055
4.22	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.99	.0346	-.0026	.0041
4.26	19.61	22.21	24.61	297.2	300.7	22.15	299.30	853.02	.0440	-.0023	.0053
4.26	19.51	22.11	24.51	297.4	300.4	22.05	299.27	852.44	.0393	-.0024	.0046
4.30	19.61	22.21	24.61	297.2	300.7	22.15	299.30	853.16	.0440	-.0023	.0055
4.30	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.85	.0346	-.0026	.0039
4.31	19.51	22.11	24.51	297.4	300.4	22.05	299.27	852.34	.0393	-.0024	.0045
4.31	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.89	.0346	-.0026	.0040
4.35	19.51	22.11	24.51	297.4	300.4	22.05	299.27	852.37	.0393	-.0024	.0045
4.35	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.71	.0346	-.0026	.0037
4.39	19.51	22.11	24.51	297.4	300.4	22.05	299.27	852.48	.0393	-.0024	.0047
4.39	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.74	.0346	-.0026	.0038
4.43	19.51	22.11	24.51	297.4	300.4	22.05	299.27	852.36	.0393	-.0024	.0045
4.44	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.68	.0346	-.0026	.0037
4.47	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.70	.0346	-.0026	.0037
4.50	19.41	22.01	24.41	297.6	300.1	21.95	299.23	851.65	.0346	-.0026	.0037

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