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對分佈於離散具值域的模之自同胎代數的同構問題研究

A study of the isomorphism problems of endomorphism algebras of modules over discrete valuation domains

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一、中文摘要

如果，已知分佈於相同環上的兩模具有同構的自同胎環，而我們能得到該二模本身亦同構的理論，我們稱之為同構理論（或者同構問題）。假設，M 和 N 是分佈於完備離散具值域 R 上的兩 torsion 模，且我們有同構的自同胎代數 End(M) 與 End(N)，則 Baer 和 Kaplansky [K54] 證明了這二模 M 和 N 本身亦為同構。Wolfson [W64] 對 torsion-free 模的情形也提出了一類似的證明。

在本計畫中，我們主要的目的是爭對分佈於完備離散具值域的任意單一模 M，考慮當預知自同胎代數 $\mathrm{End}(M) \cong \mathrm{End}(N)$ 為同構時，也能得到 M 與 N 同構的各種條件。在尋求條件的同時，我們發現了，所有分佈於完備離散具值域上模的同構理論問題，都可以化簡成 M 為 reduced 模且 M/tM 為 divisible 模的類似同構問題。也就是說，如果我們能解決後者的問題，我們也就可以解決所有的同構問題。對此，我們也提出了一種同構理論。

關鍵詞：自同胎代數、模、離散具值域、同構理論、同構問題

Abstract

An isomorphism theorem (or and isomorphism problem) is one which concludes that two modules over the same ring are isomorphic if their endomorphism rings are isomorphic. If M and N are torsion modules over a complete discrete valuation domain R such that End(M) and End(N) are isomorphic, then Baer and Kaplansky [K54] proved that M and N are isomorphic modules. Wolfson [W62] proved a similar theorem for torsion-free modules.

In this project, our primary purpose is to consider an arbitrary module M over a complete discrete valuation domain R, and search all possible conditions, under which the isomorphism $\mathrm{End}(M) \cong \mathrm{End}(N)$ will give an isomorphism from M onto N. As results, we show that the isomorphism problems of modules over a complete discrete valuation domain can be reduced to the isomorphism problems of reduced modules M with M/tM divisible, and solve one of such isomorphism problems.

Keywords: endomorphism algebra, module, discrete valuation domain, isomorphism theorem, isomorphism problem

二、緣由與目的

A *discrete valuation domain* R is a PID with exactly one prime element p. For an R-module M, where R is a discrete valuation domain, we can introduce a topology on M by taking $\{p^n M : n \in \boldsymbol{Z}, n \geq 0\}$ to be a fundamental system neighborhood of 0. A *complete R*-module M is a Hausdorff topological space in which every Cauchy sequence converges. To fix notation, throughout this report, we shall denote R a complete discrete valuation domain. If M is an R-module, the torsion submodule of M will be denoted by tM. We denote the divisible summand of M by $p^{\infty}M$. Thus $M \cong (M/p^{\infty}M) \oplus p^{\infty}M$, where $M/p^{\infty}M$ is reduced.

Given R-modules M and N, we have two associated R-algebras End(M) and End(N). Suppose there is an isomorphism $\Phi : \mathrm{End}(M) \to \mathrm{End}(N)$ between End(M) and End(N). A (strong) isomorphism theorem (or an isomorphism problem) is to conclude that Φ is induced by an isomorphism $\phi : M \to N$; i.e, $\Phi(\alpha) = \phi\alpha\phi^{-1}$ for each α in End(M).

In 1954, Kaplansky in his book, *infinite abelian group* [K54], proved an isomorphism theorem for the endomorphism algebras of torsion modules over a complete discrete valuation domain. The existence of indecomposable summands for torsion-free modules enabled Wolfson [W62] to prove a similar theorem in the torsion-free case. Kaplansky's and Wolfsan's theorems have two things in common: (1) they put conditions on both modules; (2) they required that both modules be in the same class (i.e., either both torsion or both torsion-free). There is another type of isomorphism theorem, in which the class of only one of the modules is specified. For instance, May [M78] proved the theorem: *If M is a nontorsion divisible module, then any isomorphism* $\Phi : \mathrm{End}(M) \to \mathrm{End}(N)$ *is induced by an isomorphism* $\phi : M \to N$.

In this project we wish to do two things.

1. We'll show the isomorphism problems of modules over a complete discrete valuation domain can be reduced to the isomorphism problems of reduced modules M with M/tM divisible.
2. We'll present an isomorphism theorem for a reduced module M with M/tM divisible.

三、結果與討論

Given modules M and N, suppose Φ is an isomorphism from End(M) onto End(N). By using the fact that M can be decompose as a direct sum of A and D (i.e,, $M = A \oplus D$), where A is reduced and D is divisible, we can get a decomposition $N = A^{*} \oplus D^{*}$, and obtain two isomorphisms. $\mathrm{End}(A) \cong \mathrm{End}(A^{*})$ and $\mathrm{End}(D) \cong \mathrm{End}(D^{*})$.

Next, consider the possibility that whether there is an indecomposable torsion-free summand in D [F70, Theorem 23.1], then using the proofing technique of Kaplansky's theorem [K54] and May's theorem [M78], we could divide (a long process) the isomorphism problem into several cases, and end with the result: *The solutions of the isomorphism problems for modules over complete discrete valuation domains depends on the solutions of the isomorphism problem for reduced modules M with M/tM divisible.*

我們已將本研究所獲得之主要結果整理繕寫後送交發表。在該文中，我們首先定義模的同構性質如下：

Definition: A module M over a complete discrete valuation domain R is said to have the *(strong) isomorphism property* if $\Phi : \mathrm{End}(M) \to \mathrm{End}(N)$ is an isomorphism, then there is an isomorphism $\phi : M \to N$ such that $\Phi(\alpha) = \phi\alpha\phi^{-1}$ for each α in End(M).

之後，我們建立了以下主要的同構理論：

Theorem: *Let M and N be reduced modules such that M/tM and N/tN are divisible. If N has the strong isomorphism property and M can be embedded as a summand of N, then $M \oplus N$ has the strong isomorphism property.*

四、成果自評

本計畫之成果與原先預期之結果大致完全符合。主要結果且已送交發表，尚有部份結果，經由適當的整理，也會陸續尋求期刊發表。

五、參考文獻

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附件：封面格式

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